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Dyson summation without violating Ward identities and the Goldstone-boson equivalence theorem

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Abstract:

In contrast to the conventional treatment of gauge theories, in the background-field method the Ward identities for connected Green functions are not violated by Dyson summation of self-energies in finite orders of perturbation theory. Thus, Dyson summation does not spoil gauge cancelations at high energies which are ruled by the Goldstone-boson equivalence theorem. Moreover, in the background-field method the precise formulation of the equivalence theorem in higher orders (including questions of renormalization) is simplified rendering actual calculations easier. Finally, the equivalence theorem is also formulated for the Standard Model with a non-linearly realized scalar sector and for the gauged non-linear σ-model.

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I INTRODUCTION

In many field-theoretic applications, such as the treatment of finite-width effects or running couplings, it is desirable or even mandatory to resum reducible self-energy effects. The use of Dyson resummation necessarily amounts to an incomplete inclusion of higher-order effects, i.e. of those which go beyond the inspected order of perturbation theory, in theoretical predictions. However, it is well known that in the conventional approach to gauge-field theories these higher-order effects in general violate the Ward identities which follow from gauge invariance. These rule, in particular, the gauge cancelations for longitudinally polarized gauge bosons at high energies. In order to keep theoretical uncertainties under control, it is necessary to preserve the Ward identities exactly in any finite order of perturbation theory. In this paper we show that the Ward identities are not violated by Dyson summation if the gauge theory is quantized in the framework of the background-field method (BFM).

The discussion of Ward identities is naturally connected to the investigation of the Goldstone-boson equivalence theorem (ET) [1, 2, 3, 4, 5, 6] which controls the gauge cancelations for S-matrix elements. The ET relates amplitudes for reactions involving longitudinal vector bosons at high energies to those involving the associated would-be Goldstone bosons. The gauge cancelations that occur for external longitudinal gauge bosons are absent for the corresponding scalars. As the amplitudes for external scalars are much easier to evaluate, the ET facilitates the calculation of cross-sections for reactions with longitudinal vector bosons at high energies in the Standard Model (SM) and other models. But the ET is not only a calculational tool. Because it relates longitudinal vector bosons to the Higgs sector, it might allow to derive information on the mechanism of spontaneous symmetry breaking from the experimental study of longitudinal vector bosons. In Ref. [7] it was even pointed out that the validity of the ET itself can serve as criterion to discriminate processes for probing the electroweak symmetry-breaking sector.

We start by recapitulating the formulation of the ET as described in the literature. The ET was derived in the SM for tree-level amplitudes a long time ago by Cornwall, Levin and Tiktopoulos [1] and extended to all orders in Refs. [2, 3]. The derivation of the ET consists of three basic steps:

(i) The BRS invariance of the underlying theory implies

$$\langle A|\mathcal{T} F_{a_1}(x_1)\cdots F_{a_k}(x_k)|B\rangle_{\text{connected}} = 0 \tag{1}$$

for arbitrarily many insertions of R_ξ gauge-fixing terms $F_{a_i} = \partial V_{a_i} + \xi_a c_a \phi_{a_i}$ taken between physical states $|A\rangle$ and $|B\rangle$. These identities relate the "scalar components" ∂V_{a_i} of massive vector fields V_{a_i} with their would-be Goldstone-boson partners ϕ_{a_i} for the corresponding external field points x_i.

(ii) The longitudinal polarization vector for high-energetic vector bosons with momentum k_i is given by $\varepsilon^\mu_{\mathrm{L},a_i}(k_i) = k_i^\mu/M_{a_i} + \mathcal{O}(M_{a_i}/k_i^0)$. Thus, at high energies $\varepsilon_\mathrm{L} V \approx kV \leftrightarrow \partial V$ and the identities (1) yield linear relations between Green functions for longitudinally polarized gauge bosons and their would-be Goldstone-boson partners. Amputation of the "gauge-fixing legs" yields relations between the corresponding

transition matrix elements. For the precise formulation of these relations the Ward-Takahashi identities for gauge-boson propagators have to be investigated. It turns out that in higher orders the simple tree-level form of the ET in general is modified by correction factors [4, 5, 6], which depend on the particular choice of the renormalization scheme in the (unphysical) scalar sector. Upon exploiting the Ward-Takahashi identities for the propagators, the correction factors can be expressed in terms of gauge-boson and scalar self-energies [6]. By adjusting the renormalization of the scalar sector appropriately, the correction factors can be absorbed into the renormalization constants [5, 6].

(iii) Finally, it has to be clarified to which order in k_i^0/M the relation between amplitudes involving longitudinal vector bosons and those involving would-be Goldstone bosons is valid. As far as the SM in the high-energy limit is concerned [1, 2, 3], unitarity ensures that the scalar amplitudes coincide with the corresponding longitudinal vector-boson amplitudes up to $\mathcal{O}(M/k_i^0)$, where M generically represents all particle masses present in the SM. This requires, in particular, that the energy E of the process has to exceed the Higgs-boson mass M_{H}, i.e. $E \gg M_{\mathrm{H}}$. For arbitrary M_{H} the leading powers in Higgs-boson mass M_{H} and energy E (i.e. terms of the order $E^m M_{\mathrm{H}}^n$), and thus the range of validity of the ET, can be determined by power counting for each Feynman graph as described in Refs. [8, 9].

Apart from the SM, the ET was also established for chiral Lagrangians [10] and the gauged non-linear σ-model [9] in higher orders. The validity of the ET in the case of general effective vector-boson interactions at tree level was investigated by power counting in Ref. [11].

In this paper we discuss the Ward identities for connected Green functions and the ET for higher orders within the BFM. The BFM [12, 13] allows the construction of a gauge-invariant effective action that leads to the same S matrix as the conventional effective action [14]. The resulting vertex and Green functions obey simple tree-level-like Ward identities even after renormalization [15, 16]. We derive these Ward identities for the generating functional of renormalized connected Green functions. A careful amputation procedure leads to identities for amputated Green functions which imply the ET. The absence of ghost contributions in the BFM Ward identities simplifies the precise formulation of the ET as compared to the conventional formalism. The correction factors to the naïve ET can be easily obtained from the transverse parts of the gauge-boson self-energies.

The paper is organized as follows: In Sect. II we derive the Ward identities for the generating functional of connected Green functions within the BFM and give the Ward identities for on-shell amputated, connected Green functions. In Sect. III we derive the wave-function renormalization constants needed for the calculation of S-matrix elements. The ET is derived in Sect. IV and generalized to the non-linearly realized scalar sector of the SM in Sect. V. Section VI contains a summary and conclusions. In App. A we list some conventions. In App. B we present the proof of the Ward identities relevant for the ET. Appendix C provides the explicit one-loop results for the wave-function renormalization constants.

II WARD IDENTITIES

II.1 Background-field effective action

As we will base our investigations on the BFM it is useful to sketch the essential ingredients of this approach [12]. We follow the treatment of Abbott [13]. The BFM is a technique for quantizing gauge theories that leads to a gauge-invariant effective action. To this end the usual fields $\hat{\phi}$ appearing in the classical Lagrangian $\mathcal{L}_{\rm classical}$ are decomposed into background fields $\hat{\phi}$ (marked by a hat) and quantum fields ϕ,

$$\mathcal{L}_{\rm classical}(\hat{\phi}) \to \mathcal{L}_{\rm classical}(\hat{\phi}+\phi). \tag{2}$$

While the background fields are treated as external sources, only the quantum fields are variables of integration in the path integral. The gauge-fixing term with associated (quantum) gauge parameters ξ_Q, which is added to allow for the construction of quantum-field propagators, is chosen such that the path integral is invariant with respect to gauge transformations of the background fields. By the usual Legendre transformation of the generating functional of connected Green functions with respect to the quantum fields one obtains an effective action. Putting the natural sources of this effective action, which are related to the quantum fields, to zero, one arrives at an effective action $\Gamma[\hat{\phi}]$ which only depends on background fields. This effective action is gauge-invariant, i.e. invariant under gauge transformations of the background fields, which act as sources, and leads to the same S matrix as the conventional effective action [14]. The BFM was worked out for the electroweak SM in Ref. [15]. In this reference all Feynman rules including the relevant one-loop counterterms are listed. In the following we need only the fact that a gauge-invariant background-field effective action $\Gamma[\hat{\phi}]$ exists.

The invariance of the background-field effective action Γ under background-field gauge transformations with associated group parameters $\hat{\theta}^a$, $a = A, Z, \pm$, gives rise to simple tree-level-like Ward identities:

$$\frac{\delta\Gamma}{\delta\hat{\theta}^a} = 0. \tag{3}$$

The explicit form of these functional identities was given in Ref. [17] and can also be easily inferred from the results of Ref. [15]. When appropriately renormalized [15], these Ward identities hold for the renormalized effective action as well. To this end the field renormalization constants of the gauge-boson and scalar fields must be related to the parameter renormalization constants. The latter can still be chosen arbitrarily; in particular, the usual on-shell scheme can be adopted for the parameters. However, since the renormalized fields mix on-shell and have propagators with residues different from one, a non-trivial wave-function renormalization is required when calculating S-matrix elements. In the following, all quantities have to be understood as renormalized in the way described in Ref. [15][1].

II.2 Connected Green functions

The connected Green functions and the S matrix are constructed by forming trees with vertices from Γ connected by lowest-order background-field propagators [14]. In

[1] Alternatively, one can avoid a non-trivial wave function renormalization by introducing appropriate renormalized fields if one allows for modifications of the renormalized Ward identities [18].

order to define these propagators, one has to add a gauge-fixing term for the background fields to Γ resulting in

$$\Gamma^{\mathrm{full}} = \Gamma + \mathrm{i} \int \mathrm{d}^4x \, \mathcal{L}_{\mathrm{GF}}^{\mathrm{BF}}. \tag{4}$$

We choose the usual 't Hooft gauge-fixing term

$$\mathcal{L}_{\mathrm{GF}}^{\mathrm{BF}} = -\frac{1}{2\hat{\xi}_A}\left(\partial\hat{A}\right)^2 - \frac{1}{2\hat{\xi}_Z}\left(\partial\hat{Z} - \hat{\xi}_Z M_{\mathrm{Z}}\hat{\chi}\right)^2 - \frac{1}{\hat{\xi}_W}\left|\partial\hat{W}^+ - \mathrm{i}\hat{\xi}_W M_{\mathrm{W}}\hat{\phi}^+\right|^2 \tag{5}$$

with the background-gauge-fixing parameters $\hat{\xi}_A, \hat{\xi}_Z, \hat{\xi}_W$. Our conventions for fields, vertex functions, etc. follow the ones of Ref. [15] throughout.

From the Ward identities (3) for Γ one obtains the Ward identities for Γ^{full}:

$$\frac{\delta\Gamma^{\mathrm{full}}}{\delta\hat{\theta}^a} = \frac{\delta}{\delta\hat{\theta}^a}\mathrm{i}\int \mathrm{d}^4x \, \mathcal{L}_{\mathrm{GF}}^{\mathrm{BF}}. \tag{6}$$

The left-hand side is formally identical to the left-hand side of (3) with Γ replaced by Γ^{full}, and thus can be simply read off from Ref. [17]. The right-hand side can be evaluated from the behavior of the fields in (5) under background gauge transformations [15] resulting in

$$\begin{aligned}
\frac{\delta}{\delta\hat{\theta}^A}\,\mathrm{i}\int \mathrm{d}^4x\mathcal{L}_{\mathrm{GF}}^{\mathrm{BF}} &= -\frac{\mathrm{i}}{\hat{\xi}_A}\Box\partial\hat{A} + \sum_{\pm}\frac{(\pm e)}{\hat{\xi}_W}\left(\hat{W}_\mu^{\pm}\partial^\mu \pm \mathrm{i}\hat{\xi}_W M_{\mathrm{W}}\hat{\phi}^{\pm}\right)\left(\partial\hat{W}^{\mp} \pm \mathrm{i}\hat{\xi}_W M_{\mathrm{W}}\hat{\phi}^{\mp}\right), \\
\frac{\delta}{\delta\hat{\theta}^Z}\,\mathrm{i}\int \mathrm{d}^4x\mathcal{L}_{\mathrm{GF}}^{\mathrm{BF}} &= -\frac{\mathrm{i}}{\hat{\xi}_Z}\left(\Box + \hat{\xi}_Z M_{\mathrm{Z}}^2\right)\left(\partial\hat{Z} - \hat{\xi}_Z M_{\mathrm{Z}}\hat{\chi}\right) - \frac{\mathrm{i}eM_{\mathrm{Z}}}{2c_{\mathrm{W}}s_{\mathrm{W}}}\hat{H}\left(\partial\hat{Z} - \hat{\xi}_Z M_{\mathrm{Z}}\hat{\chi}\right) \\
&\quad - \sum_{\pm}\frac{(\pm e)}{\hat{\xi}_W}\left(\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\hat{W}_\mu^{\pm}\partial^\mu \pm \mathrm{i}\hat{\xi}_W M_{\mathrm{W}}\frac{c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2}{2c_{\mathrm{W}}s_{\mathrm{W}}}\hat{\phi}^{\pm}\right)\left(\partial\hat{W}^{\mp} \pm \mathrm{i}\hat{\xi}_W M_{\mathrm{W}}\hat{\phi}^{\mp}\right), \\
\frac{\delta}{\delta\hat{\theta}^{\pm}}\,\mathrm{i}\int \mathrm{d}^4x\mathcal{L}_{\mathrm{GF}}^{\mathrm{BF}} &= -\frac{\mathrm{i}}{\hat{\xi}_W}\left(\Box + \hat{\xi}_W M_{\mathrm{W}}^2\right)\left(\partial\hat{W}^{\mp} \pm \mathrm{i}\hat{\xi}_W M_{\mathrm{W}}\hat{\phi}^{\mp}\right) \\
&\quad \mp \frac{e}{\hat{\xi}_W}\left[\left(\hat{A}_\mu - \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\hat{Z}_\mu\right)\partial^\mu \pm \mathrm{i}\hat{\xi}_W\frac{M_{\mathrm{W}}}{2s_{\mathrm{W}}}\left(\hat{H} \pm \mathrm{i}\hat{\chi}\right)\right]\left(\partial\hat{W}^{\mp} \pm \mathrm{i}\hat{\xi}_W M_{\mathrm{W}}\hat{\phi}^{\mp}\right) \\
&\quad \pm \frac{e}{\hat{\xi}_A}\hat{W}_\mu^{\mp}\partial^\mu\partial\hat{A} \mp \frac{e}{\hat{\xi}_Z}\left(\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\hat{W}_\mu^{\mp}\partial^\mu \mp \mathrm{i}\hat{\xi}_Z\frac{M_{\mathrm{Z}}}{2s_{\mathrm{W}}}\hat{\phi}^{\mp}\right)\left(\partial\hat{Z} - \hat{\xi}_Z M_{\mathrm{Z}}\hat{\chi}\right),
\end{aligned} \tag{7}$$

where $c_{\mathrm{W}} = M_{\mathrm{W}}/M_{\mathrm{Z}}$, $s_{\mathrm{W}}^2 = 1 - c_{\mathrm{W}}^2$, and e denotes the elementary charge.

The generating functional of connected Green functions, Z_{c}, is obtained from Γ^{full} as usual by a Legendre transformation,

$$Z_{\mathrm{c}}[J_{\hat{F}}, J_f, J_{\bar{f}}] = \Gamma^{\mathrm{full}}[\hat{F}, f, \bar{f}] + \mathrm{i}\int \mathrm{d}^4x\Big[\sum_{\hat{F}} J_{\hat{F}^\dagger}\hat{F} + \sum_f(\bar{f}J_f + J_{\bar{f}}f)\Big] \tag{8}$$

with $\hat{F} = \hat{A}, \hat{Z}, \hat{W}^+, \hat{W}^-, \hat{H}, \hat{\chi}, \hat{\phi}^+, \hat{\phi}^-$ and

$$\mathrm{i}J_{\hat{F}^\dagger} = -\frac{\delta\Gamma^{\mathrm{full}}}{\delta\hat{F}}, \qquad \mathrm{i}J_{\bar{f}} = \frac{\delta\Gamma^{\mathrm{full}}}{\delta f}, \qquad \mathrm{i}J_f = -\frac{\delta\Gamma^{\mathrm{full}}}{\delta\bar{f}}, \tag{9}$$

and conversely

$$\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat F^\dagger}} = \hat F, \qquad \frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\bar f}} = f, \qquad \frac{\delta Z_{\rm c}}{{\rm i}\delta J_{f}} = -\bar f. \tag{10}$$

The field $\hat F^\dagger$ denotes the complex conjugate of $\hat F$, i.e. for instance $\hat A^\dagger = \hat A$ but $(\hat W^+)^\dagger = \hat W^-$.

As a consequence, the 1-particle reducible Green functions and S-matrix elements are composed as in the conventional formalism from a tree structure of vertex functions. While the vertices in these trees are directly given by the background-field vertex functions, the propagators are determined as the inverse of the two-point-vertex functions resulting from $\Gamma^{\rm full}$.

Inserting (9) and (10) into (6) and (7), we find the Ward identities for the generating functional of connected Green functions in the BFM,

$$\begin{aligned}
&{\rm i}\partial J_{\hat A} + \sum_\pm (\pm e)\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat W^\pm}} J^\mu_{\hat W^\pm} + \sum_\pm (\pm e)\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\phi^\pm}} J_{\hat\phi^\pm} - e\sum_f Q_f\left(\frac{\delta Z_{\rm c}}{{\rm i}\delta J_f}J_f + J_{\bar f}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\bar f}}\right)\\
&\quad = -\frac{{\rm i}}{\hat\xi_A}\Box\partial^\mu\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat A}} + \sum_\pm\frac{(\pm e)}{\hat\xi_W}\left(\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat W^\mp}}\partial^\mu \pm {\rm i}\hat\xi_W M_{\rm W}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\phi^\mp}}\right)\left(\partial^\nu\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\nu_{\hat W^\pm}} \pm {\rm i}\hat\xi_W M_{\rm W}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\phi^\pm}}\right),\\[2ex]
&{\rm i}\partial J_{\hat Z} + {\rm i}M_{\rm Z}J_{\hat\chi} - \sum_\pm(\pm e)\frac{c_{\rm W}}{s_{\rm W}}\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat W^\pm}}J^\mu_{\hat W^\pm} - \sum_\pm(\pm e)\frac{c_{\rm W}^2-s_{\rm W}^2}{2c_{\rm W}s_{\rm W}}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\phi^\pm}}J_{\hat\phi^\pm}\\
&\qquad + \frac{{\rm i}e}{2c_{\rm W}s_{\rm W}}\left(\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat H}}J_{\hat\chi} - \frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\chi}}J_{\hat H}\right) + e\sum_f\left(\frac{\delta Z_{\rm c}}{{\rm i}\delta J_f}(v_f + a_f\gamma_5)J_f + J_{\bar f}(v_f - a_f\gamma_5)\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\bar f}}\right)\\
&\quad = -\frac{{\rm i}}{\hat\xi_Z}(\Box + \hat\xi_Z M_Z^2)\left(\partial^\mu\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat Z}} - \hat\xi_Z M_{\rm Z}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\chi}}\right) - \frac{{\rm i}eM_{\rm Z}}{2c_{\rm W}s_{\rm W}}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat H}}\left(\partial^\mu\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat Z}} - \hat\xi_Z M_{\rm Z}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\chi}}\right)\\
&\qquad - \sum_\pm\frac{(\pm e)}{\hat\xi_W}\left(\frac{c_{\rm W}}{s_{\rm W}}\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat W^\mp}}\partial^\mu \pm {\rm i}\hat\xi_W M_{\rm W}\frac{c_{\rm W}^2-s_{\rm W}^2}{2c_{\rm W}s_{\rm W}}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\phi^\mp}}\right)\left(\partial^\nu\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\nu_{\hat W^\pm}} \pm {\rm i}\hat\xi_W M_{\rm W}\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat\phi^\pm}}\right),\\[2ex]
&{\rm i}\partial J_{\hat W^\mp} \pm M_{\rm W}J_{\hat\phi^\mp} \mp e\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat W^\pm}}\left(J^\mu_{\hat A} - \frac{c_{\rm W}}{s_{\rm W}}J^\mu_{\hat Z}\right) \pm e\left(\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat A}} - \frac{c_{\rm W}}{s_{\rm W}}\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat Z}}\right)J^\mu_{\hat W^\mp}\\
&\qquad \mp \frac{e}{2s_{\rm W}}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\phi^\pm}}(J_{\hat H} \pm {\rm i}J_{\hat\chi}) \pm \frac{e}{2s_{\rm W}}\left(\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat H}} \pm {\rm i}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\chi}}\right)J_{\hat\phi^\mp}\\
&\qquad + \frac{e}{\sqrt2 s_{\rm W}}\sum_{(f_+,f_-)}\left(\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{f_\pm}}\frac{1+\gamma_5}{2}J_{f_\mp} + J_{\bar f_\pm}\frac{1-\gamma_5}{2}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\bar f_\mp}}\right)\\
&\quad = -\frac{{\rm i}}{\hat\xi_W}(\Box + \hat\xi_W M_{\rm W}^2)\left(\partial^\mu\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat W^\pm}} \pm {\rm i}\hat\xi_W M_{\rm W}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\phi^\pm}}\right)\\
&\qquad \mp \frac{e}{\hat\xi_W}\left[\left(\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat A}} - \frac{c_{\rm W}}{s_{\rm W}}\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat Z}}\right)\partial^\mu \pm {\rm i}\hat\xi_W\frac{M_{\rm W}}{2s_{\rm W}}\left(\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat H}} \pm {\rm i}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\chi}}\right)\right]\\
&\qquad\quad \times\left(\partial^\nu\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\nu_{\hat W^\pm}} \pm {\rm i}\hat\xi_W M_{\rm W}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\phi^\pm}}\right) \pm \frac{e}{\hat\xi_A}\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat W^\pm}}\partial^\mu\partial^\nu\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\nu_{\hat A}}\\
&\qquad \mp \frac{e}{\hat\xi_Z}\left(\frac{c_{\rm W}}{s_{\rm W}}\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\mu_{\hat W^\pm}}\partial^\mu \mp {\rm i}\hat\xi_Z\frac{M_{\rm Z}}{2s_{\rm W}}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\phi^\pm}}\right)\left(\partial^\nu\frac{\delta Z_{\rm c}}{{\rm i}\delta J^\nu_{\hat Z}} - \hat\xi_Z M_{\rm Z}\frac{\delta Z_{\rm c}}{{\rm i}\delta J_{\hat\chi}}\right),
\end{aligned} \tag{11}$$

where $v_f = (I^3_{\mathrm{W},f} - 2s^2_{\mathrm{W}} Q_f)/(2s_{\mathrm{W}}c_{\mathrm{W}})$ and $a_f = I^3_{\mathrm{W},f}/(2s_{\mathrm{W}}c_{\mathrm{W}})$ are the vector and axial-vector couplings of the Z boson to the fermion f with relative charge Q_f and third component of weak iso-spin $I^3_{\mathrm{W},f}$. In (11) $f_\pm$ denote the fermionic iso-spin partners with iso-spins $\pm 1/2$, and the sum over (f_+, f_-) runs over all iso-spin doublets.

By taking functional derivatives of (11) with respect to the sources one obtains Ward identities for connected Green functions. These Ward identities hold for any fixed loop order in perturbation theory exactly. This is evident if one expands everything including propagators in powers of the coupling constant e. However, *the Ward identities hold as well after Dyson summation of the self-energy corrections if the inverse propagators, which are just the two-point vertex functions, are calculated in the same loop order as all other vertex functions.* In order to see this, one has to go back to the background-field effective action Γ and its Ward identities (3). As these are linear in Γ, its n-loop approximation $\Gamma|_{\mathrm{n-loop}}$ fulfills exactly the same Ward identities. Substituting $\Gamma|_{\mathrm{n-loop}}$ instead of Γ into (4) analogously defines $\Gamma^{\mathrm{full}}|_{\mathrm{n-loop}}$, which in turn defines $Z_{\mathrm{c}}|_{\mathrm{n-loop}}$ via a Legendre transformation, as written down for Z_{c} in (8)–(10). Consequently, $Z_{\mathrm{c}}|_{\mathrm{n-loop}}$ is the generating functional for connected Green functions built of vertex functions in n-loop approximation and propagators that are defined as the inverse two-point vertex functions in the same approximation, i.e. all propagators include the Dyson-resummed self-energies in n-loop approximation. By construction all previous relations remain valid if Γ, Γ^{full}, and Z_{c} are replaced by $\Gamma|_{\mathrm{n-loop}}$, $\Gamma^{\mathrm{full}}|_{\mathrm{n-loop}}$, and $Z_{\mathrm{c}}|_{\mathrm{n-loop}}$, respectively. This proves, in particular, that $Z_{\mathrm{c}}|_{\mathrm{n-loop}}$, which involves Dyson summation, fulfills the Ward identities (11) exactly.

Consequently, Dyson summation within the BFM does not disturb the high-energy behavior of physical amplitudes; in particular, gauge cancelations are not violated. This feature is not present in the conventional formalism. We note that the BFM vertex functions still depend on the quantum gauge parameter ξ_Q. However, the logarithmic contributions to the self-energies that dominate at high energies are gauge-independent and universal [19]; they are in fact governed by the renormalization group.

The previous considerations show that the BFM allows Dyson summation and thus, in particular, the introduction of finite-width effects without spoiling the Ward identities. Unfortunately, a dependence on the quantum gauge parameter remains, which cannot be fixed on physical grounds. So far—to the best of our knowledge—there is no prescription available that yields a unique unambiguous answer in the general case. However, for particles that decay only into fermions, such as the Z and W bosons, a practical way consists in resumming only the fermionic one-loop corrections [20]. Since these are identical in the BFM and the conventional approach, and the complete one-loop corrections are just the sum of the fermionic and bosonic corrections, our analysis provides an independent proof of the fermion-loop scheme for the treatment of finite width-effects in tree-level amplitudes.

II.3 Propagators

In the following, we need the explicit form of the Ward identities for the two-point functions, i.e. the propagators, of the gauge and scalar bosons. These are obtained by differentiating (11) with respect to the corresponding sources, putting all sources to zero and using $\delta Z_{\mathrm{c}}/\delta J_{\hat{F}}|_{J_{\hat{F}}=0} = 0$. Whereas this last equation is clear for all other fields it is

enforced for the physical Higgs field by a renormalization condition (vanishing tadpole). Introducing a 4×4 matrix

$$G^0_{(\mu\nu)} = \begin{pmatrix} G^{\hat{A}\hat{A}}_{\mu\nu} & G^{\hat{A}\hat{Z}}_{\mu\nu} & G^{\hat{A}\hat{\chi}}_{\mu} & G^{\hat{A}\hat{H}}_{\mu} \\ G^{\hat{Z}\hat{A}}_{\mu\nu} & G^{\hat{Z}\hat{Z}}_{\mu\nu} & G^{\hat{Z}\hat{\chi}}_{\mu} & G^{\hat{Z}\hat{H}}_{\mu} \\ G^{\hat{\chi}\hat{A}}_{\nu} & G^{\hat{\chi}\hat{Z}}_{\nu} & G^{\hat{\chi}\hat{\chi}} & G^{\hat{\chi}\hat{H}} \\ G^{\hat{H}\hat{A}}_{\nu} & G^{\hat{H}\hat{Z}}_{\nu} & G^{\hat{H}\hat{\chi}} & G^{\hat{H}\hat{H}} \end{pmatrix} \tag{12}$$

for the neutral boson propagators and a 2×2 matrix

$$G^{\pm}_{(\mu\nu)} = \begin{pmatrix} G^{\hat{W}^{\pm}\hat{W}^{\mp}}_{\mu\nu} & G^{\hat{W}^{\pm}\hat{\phi}^{\mp}}_{\mu} \\ G^{\hat{\phi}^{\pm}\hat{W}^{\mp}}_{\nu} & G^{\hat{\phi}^{\pm}\hat{\phi}^{\mp}} \end{pmatrix} \tag{13}$$

for the charged boson propagators, the Ward identities can be compactly written as

$$\begin{aligned} (k^\mu, 0, 0, 0)G^0_{(\mu\nu)} &= -\mathrm{i}\hat{\xi}_A \frac{1}{k^2}(k_\nu, 0, 0, 0), \\ (0, k^\mu, \mathrm{i}\hat{\xi}_Z M_\mathrm{Z}, 0)G^0_{(\mu\nu)} &= -\mathrm{i}\hat{\xi}_Z \frac{1}{k^2 - \hat{\xi}_Z M_\mathrm{Z}^2}(0, k_\nu, -\mathrm{i}M_\mathrm{Z}, 0), \\ (k^\mu, \pm\hat{\xi}_W M_\mathrm{W})G^{\pm}_{(\mu\nu)} &= -\mathrm{i}\hat{\xi}_W \frac{1}{k^2 - \hat{\xi}_W M_\mathrm{W}^2}(k_\nu, \mp M_\mathrm{W}), \end{aligned} \tag{14}$$

where we have turned to momentum space for later convenience. In (14) k is the momentum flowing through the two-point functions. Our conventions for the Fourier transformation from coordinate to momentum space are summarized in App. A.

Note that in the conventional formalism the left-hand sides of these relations are much more complicated and involve ghost contributions [5, 6].

II.4 Amputated connected Green functions

The Ward identities (11) involve four different types of terms: The effective action gives rise to terms involving J or $J\delta Z_\mathrm{c}/\delta J$, the gauge-fixing term introduces terms containing $\delta Z_\mathrm{c}/\delta J$ or $(\delta Z_\mathrm{c}/\delta J)^2$. The J terms obviously drop out when more than one functional derivative is taken, i.e. they merely contribute to the Ward identities for two-point functions, which have been given in the previous section. In App. B we proof that the $J\delta Z_\mathrm{c}/\delta J$ and $(\delta Z_\mathrm{c}/\delta J)^2$ terms do not contribute to Green functions after amputating and putting all external physical fields on their mass shell. Consequently, the Ward identities (11) imply

$$\begin{aligned} 0 &= k^\mu \frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J^\mu_{\hat{A}}} + \text{ o.v.t.}\,, \\ 0 &= \left(k^\mu \frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J^\mu_{\hat{Z}}} + \mathrm{i}\hat{\xi}_Z M_\mathrm{Z} \frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\hat{\chi}}}\right) + \text{ o.v.t.}\,, \\ 0 &= \left(k^\mu \frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J^\mu_{\hat{W}^\pm}} \pm \hat{\xi}_W M_\mathrm{W} \frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\hat{\phi}^\pm}}\right) + \text{ o.v.t.}\,, \end{aligned} \tag{15}$$

where o.v.t. (on-shell vanishing terms) stands for terms that vanish after taking derivatives with respect to physical fields and subsequent amputation and on-shell projection.

Equations (15) represent the identity (1) for one gauge-fixing condition. The generalization to more external "gauge-fixing legs" is also shown in App. B.

Note that the Ward identities (15) and their generalizations are identical to the identities (1) of the conventional formalism. This is due to the fact that they hold for on-shell physical fields and that the gauge-fixing term for the background fields is identical to the one in the conventional formalism.

In order to derive the ET from (15), we have to amputate the external legs that correspond to the gauge-fixing operators. Because of the mixing between longitudinal gauge bosons and would-be Goldstone bosons, this amputation must be done carefully. Marking amputated external legs by lowering the corresponding field index of the Green function, we can write the relation between non-amputated and amputated vertex functions for neutral bosons as follows

$$\begin{pmatrix} G^{\hat{A}\dots}_{\dots,\mu} \\ G^{\hat{Z}\dots}_{\dots,\mu} \\ G^{\hat{\chi}\dots}_{\dots} \\ G^{\hat{H}\dots}_{\dots} \end{pmatrix} = G^{0}_{(\mu\nu)} \begin{pmatrix} G^{\dots,\nu}_{\hat{A}\dots} \\ G^{\dots,\nu}_{\hat{Z}\dots} \\ G^{\dots}_{\hat{\chi}\dots} \\ G^{\dots}_{\hat{H}\dots} \end{pmatrix}, \qquad \begin{pmatrix} G^{\hat{W}^{\pm}\dots}_{\dots\mu} \\ G^{\hat{\phi}^{\pm}\dots}_{\dots} \end{pmatrix} = G^{\pm}_{(\mu\nu)} \begin{pmatrix} G^{\dots,\nu}_{\hat{W}^{\pm}\dots} \\ G^{\dots}_{\hat{\phi}^{\pm}\dots} \end{pmatrix}, \tag{16}$$

where the dots indicate the remaining amputated and non-amputated external legs.

In this matrix notation the Ward identities (15) read

$$(k^{\mu},0,0,0) \begin{pmatrix} G^{\hat{A}}_{\dots,\mu} \\ G^{\hat{Z}}_{\dots,\mu} \\ G^{\hat{\chi}}_{\dots} \\ G^{\hat{H}}_{\dots} \end{pmatrix} = 0, \qquad (0,k^{\mu},\mathrm{i}\hat{\xi}_{Z}M_{\mathrm{Z}},0) \begin{pmatrix} G^{\hat{A}}_{\dots,\mu} \\ G^{\hat{Z}}_{\dots,\mu} \\ G^{\hat{\chi}}_{\dots} \\ G^{\hat{H}}_{\dots} \end{pmatrix} = 0,$$

$$(0,k^{\mu},\pm\hat{\xi}_{W}M_{\mathrm{W}},0) \begin{pmatrix} G^{\hat{W}^{\pm}}_{\dots,\mu} \\ G^{\hat{\phi}^{\pm}}_{\dots} \end{pmatrix} = 0, \tag{17}$$

assuming that all physical external legs are already amputated and put on-shell. Upon inserting (16) into (17) and using (14), we obtain the Ward identities for on-shell amputated Green functions,

$$k^{\nu}G_{\hat{A}\dots,\nu} = 0, \qquad k^{\nu}G_{\hat{Z}\dots,\nu} = \mathrm{i}M_{\mathrm{Z}}G_{\hat{\chi}\dots}, \qquad k^{\nu}G_{\hat{W}^{\pm}\dots,\nu} = \pm M_{\mathrm{W}}G_{\hat{\phi}^{\pm}\dots}. \tag{18}$$

The corresponding Ward identities in the conventional formalism involve extra factors, which depend on renormalization constants and unphysical gauge-boson and would-be Goldstone-boson self-energies. These factors can be eliminated by suitably tuning the renormalization in the unphysical sector [5, 6]. In the BFM these factors are naturally absent owing to the background-field gauge invarinace.

The first of the Ward identities (18) expresses transversality for on-shell photons, the other two imply the ET for the massive gauge bosons, as will be described in Sect. IV.

The Ward identities for arbitrarily many gauge-fixing legs follow from (B10) and (16) exactly in the same way.

III WAVE-FUNCTION RENORMALIZATION

As already mentioned, the background fields are assumed to be renormalized as described in Ref. [15]. This choice implies that the field renormalization constants are adjusted to the parameter renormalization constants such that unrenormalized and renormalized Ward identities are formally identical. However, it also implies that—except for the photon—the residues of the propagators differ from one, and that the (on-shell) Z-boson field mixes with the photon field at $k^2 = M_Z^2$. Consequently, we have to carry out a (UV-finite) wave-function renormalization when constructing S-matrix elements from on-shell amputated Green functions.[2]

In the charged sector the situation is relatively simple, since the physical components of the W-boson field $\hat{W}^{\pm}$ do not mix with any other field. The mixing with the fields $\hat{\phi}^{\pm}$ only takes place for the unphysical components of $\hat{W}^{\pm}$. An S-matrix element involving an external W boson can only differ in normalization from the corresponding on-shell amputated Green function which is contracted with the polarization vector $\varepsilon_{\mathrm{W}}(k)$ of the external W boson,

$$\langle \ldots | S | W^{\pm}(k) \ldots \rangle = R_{\hat{W}}^{1/2} \varepsilon_{\mathrm{W},\mu}(k) G^{\mu}_{\hat{W}^{\pm}\ldots}(k, \ldots). \tag{19}$$

The wave-function renormalization constant $R_{\hat{W}}$ is fixed by requiring that the pole of the transverse part of the W-boson propagator has residue one, or equivalently

$$R_{\hat{W}} \,\mathrm{Re} \left\{ \frac{\mathrm{i}\Gamma^{\hat{W}^+\hat{W}^-}_{\mu\nu}(k)}{k^2 - M_{\mathrm{W}}^2} \right\} \varepsilon^{\nu}_{\mathrm{W}}(k) \Bigg|_{k^2 = M_{\mathrm{W}}^2} = \varepsilon_{\mathrm{W},\mu}(k). \tag{20}$$

Using the decomposition of two-point functions into transverse and longitudinal parts,

$$\Gamma^{\hat{V}\hat{V}'}_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Gamma^{\hat{V}\hat{V}'}_{\mathrm{T}}(k^2) + \frac{k_\mu k_\nu}{k^2} \Gamma^{\hat{V}\hat{V}'}_{\mathrm{L}}(k^2), \tag{21}$$

the condition (20) for $R_{\hat{W}}$ implies

$$R_{\hat{W}}^{-1} = \mathrm{Re} \left\{ \frac{\partial}{\partial k^2} \mathrm{i}\Gamma^{\hat{W}^+\hat{W}^-}_{\mathrm{T}}(k^2) \right\} \Bigg|_{k^2 = M_{\mathrm{W}}^2} = \mathrm{Re}\{\mathrm{i}\Gamma'^{\hat{W}^+\hat{W}^-}_{\mathrm{T}}(M_{\mathrm{W}}^2)\}. \tag{22}$$

We remind the reader that all quantities are renormalized and that, in particular, the poles of the propagators are at the physical masses.

In the neutral sector things are complicated by the mixing between the photon and the Z boson. The mixing with the scalar fields $\hat{H}$, $\hat{\chi}$ again only takes place in the unphysical degrees of freedom and need not to be considered. The S-matrix elements involving a photon or a Z boson result from superpositions of the corresponding amputated Green functions,

$$\begin{aligned} \langle \ldots | S | A(k) \ldots \rangle &= \varepsilon_{\mathrm{A},\mu}(k) \Big[R^{1/2}_{\hat{A}\hat{A}} G^{\mu}_{\hat{A}\ldots}(k, \ldots) + R^{1/2}_{\hat{Z}\hat{A}} G^{\mu}_{\hat{Z}\ldots}(k, \ldots) \Big], \\ \langle \ldots | S | Z(k) \ldots \rangle &= \varepsilon_{\mathrm{Z},\mu}(k) \Big[R^{1/2}_{\hat{A}\hat{Z}} G^{\mu}_{\hat{A}\ldots}(k, \ldots) + R^{1/2}_{\hat{Z}\hat{Z}} G^{\mu}_{\hat{Z}\ldots}(k, \ldots) \Big]. \end{aligned} \tag{23}$$

[2]The necessity of a (UV-finite) wave-function renormalization in addition to a field renormalization which removes the UV divergences is not a peculiarity of the BFM. It arises for instance also in the minimal renormalization scheme of the SM, where multiplets of fields are renormalized by a single field renormalization constant [21].

The generalization to more gauge bosons is obvious. The wave-function renormalization constants $R_{\hat{V}\hat{V}'}$ are fixed by requiring that one-particle states are normalized and propagate without mixing with other fields. More explicitly, this means that the matrix propagator for the photon and Z-boson fields is diagonal at $k^2 = 0$ and $k^2 = M_\mathrm{Z}^2$, and the corresponding residues at the propagator poles are equal to one. For the amputated Green functions or vertex functions these conditions read

$$\begin{aligned}
\varepsilon_{\mathrm{A},\mu}(k) &= \mathrm{Re}\left\{\frac{\mathrm{i}}{k^2}\left[R_{\hat{A}\hat{A}}\Gamma^{\hat{A}\hat{A}}_{\mu\nu}(k) + 2R^{1/2}_{\hat{A}\hat{A}}R^{1/2}_{\hat{Z}\hat{A}}\Gamma^{\hat{Z}\hat{A}}_{\mu\nu}(k) + R_{\hat{Z}\hat{A}}\Gamma^{\hat{Z}\hat{Z}}_{\mu\nu}(k)\right]\right\}\varepsilon^\nu_\mathrm{A}(k)\Big|_{k^2=0},\\
0 &= \mathrm{Re}\left\{\mathrm{i}\left[R^{1/2}_{\hat{A}\hat{A}}R^{1/2}_{\hat{A}\hat{Z}}\Gamma^{\hat{A}\hat{A}}_{\mu\nu}(k) + \left(R^{1/2}_{\hat{A}\hat{A}}R^{1/2}_{\hat{Z}\hat{Z}} + R^{1/2}_{\hat{A}\hat{Z}}R^{1/2}_{\hat{Z}\hat{A}}\right)\Gamma^{\hat{Z}\hat{A}}_{\mu\nu}(k)\right.\right.\\
&\qquad\left.\left. + R^{1/2}_{\hat{Z}\hat{Z}}R^{1/2}_{\hat{Z}\hat{A}}\Gamma^{\hat{Z}\hat{Z}}_{\mu\nu}(k)\right]\right\}\varepsilon^\nu_{\mathrm{A,Z}}(k)\Big|_{k^2=0,M_\mathrm{Z}^2},\\
\varepsilon_{\mathrm{Z},\mu}(k) &= \mathrm{Re}\left\{\frac{\mathrm{i}}{k^2-M_\mathrm{Z}^2}\left[R_{\hat{A}\hat{Z}}\Gamma^{\hat{A}\hat{A}}_{\mu\nu}(k) + 2R^{1/2}_{\hat{A}\hat{Z}}R^{1/2}_{\hat{Z}\hat{Z}}\Gamma^{\hat{Z}\hat{A}}_{\mu\nu}(k) + R_{\hat{Z}\hat{Z}}\Gamma^{\hat{Z}\hat{Z}}_{\mu\nu}(k)\right]\right\}\varepsilon^\nu_\mathrm{Z}(k)\Big|_{k^2=M_\mathrm{Z}^2}.
\end{aligned} \tag{24}$$

Inserting the decomposition (21) for the two-point functions into (24), the constants $R_{\hat{V}\hat{V}'}$ can be expressed in terms of the transverse parts $\Gamma^{\hat{V}\hat{V}'}_\mathrm{T}$. Using in addition the equations

$$\Gamma^{\hat{A}\hat{A}}_\mathrm{T}(0) = \Gamma^{\hat{A}\hat{Z}}_\mathrm{T}(0) = 0, \qquad \Gamma'^{\hat{A}\hat{A}}_\mathrm{T}(0) = -\mathrm{i}, \tag{25}$$

which follow from the Ward identities and the on-shell renormalization condition for the electric charge [15], we find

$$\begin{aligned}
R^{1/2}_{\hat{A}\hat{A}} &= 1, \qquad R^{1/2}_{\hat{A}\hat{Z}} = -\frac{\mathrm{Re}\{\mathrm{i}\Gamma^{\hat{A}\hat{Z}}_\mathrm{T}(M_\mathrm{Z}^2)\}}{\mathrm{Re}\{\mathrm{i}\Gamma^{\hat{A}\hat{A}}_\mathrm{T}(M_\mathrm{Z}^2)\}}R^{1/2}_{\hat{Z}\hat{Z}},\\
R^{1/2}_{\hat{Z}\hat{A}} &= 0, \qquad R^{-1}_{\hat{Z}\hat{Z}} = \mathrm{Re}\{\mathrm{i}\Gamma'^{\hat{Z}\hat{Z}}_\mathrm{T}(M_\mathrm{Z}^2)\} - 2\,\mathrm{Re}\{\mathrm{i}\Gamma'^{\hat{A}\hat{Z}}_\mathrm{T}(M_\mathrm{Z}^2)\}\frac{\mathrm{Re}\{\mathrm{i}\Gamma^{\hat{A}\hat{Z}}_\mathrm{T}(M_\mathrm{Z}^2)\}}{\mathrm{Re}\{\mathrm{i}\Gamma^{\hat{A}\hat{A}}_\mathrm{T}(M_\mathrm{Z}^2)\}}\\
&\qquad + \mathrm{Re}\{\mathrm{i}\Gamma'^{\hat{A}\hat{A}}_\mathrm{T}(M_\mathrm{Z}^2)\}\left(\frac{\mathrm{Re}\{\mathrm{i}\Gamma^{\hat{A}\hat{Z}}_\mathrm{T}(M_\mathrm{Z}^2)\}}{\mathrm{Re}\{\mathrm{i}\Gamma^{\hat{A}\hat{A}}_\mathrm{T}(M_\mathrm{Z}^2)\}}\right)^2.
\end{aligned} \tag{26}$$

This shows, in particular, that, as a consequence of the Ward identities (25), the renormalized photon has residue one and does not mix with the Z boson for $k^2 = 0$.

We note in passing that the Z-boson mass is fixed by the condition

$$0 = \mathrm{Re}\left\{\mathrm{i}\Gamma^{\hat{Z}\hat{Z}}_\mathrm{T}(M_\mathrm{Z}^2) - \mathrm{i}\left(\Gamma^{\hat{A}\hat{Z}}_\mathrm{T}(M_\mathrm{Z}^2)\right)^2/\Gamma^{\hat{A}\hat{A}}_\mathrm{T}(M_\mathrm{Z}^2)\right\}, \tag{27}$$

which is invariant under the transformation related to the wave-function renormalization.

Decomposing the transverse parts of the two-point functions into lowest-order contributions and transverse self-energies,

$$\Gamma^{\hat{V}\hat{V}'}_\mathrm{T}(k^2) = -\mathrm{i}(k^2 - M_\mathrm{V}^2)\delta_{\hat{V}\hat{V}'} - \mathrm{i}\Sigma^{\hat{V}\hat{V}'}_\mathrm{T}(k^2), \tag{28}$$

yields simple one-loop expressions for the R factors for external W and Z bosons,

$$
\begin{aligned}
R^{1/2}_{\hat{W}} &= 1 - \frac{1}{2}\,\mathrm{Re}\left\{\Sigma'^{\hat{W}^+\hat{W}^-}_{\mathrm{T}}(M_{\mathrm{W}}^2)\right\} + \mathcal{O}(\alpha^2),\\
R^{1/2}_{\hat{Z}\hat{Z}} &= 1 - \frac{1}{2}\,\mathrm{Re}\left\{\Sigma'^{\hat{Z}\hat{Z}}_{\mathrm{T}}(M_{\mathrm{Z}}^2)\right\} + \mathcal{O}(\alpha^2),\\
R^{1/2}_{\hat{A}\hat{Z}} &= -\frac{1}{M_{\mathrm{Z}}^2}\,\mathrm{Re}\left\{\Sigma^{\hat{A}\hat{Z}}_{\mathrm{T}}(M_{\mathrm{Z}}^2)\right\} + \mathcal{O}(\alpha^2).
\end{aligned}
\tag{29}
$$

We recall that BFM vertex functions, and thus also the R factors, depend on the quantum gauge parameter ξ_Q, which enters by fixing the gauge of the quantum fields. Of course, ξ_Q cancels in any complete loop order when calculating S-matrix elements. The explicit one-loop results for the self-energies needed for the R factors according to (29) are given in App. C in 't Hooft–Feynman gauge ($\xi_Q = 1$). Finally, we note that $R^{1/2}_{\hat{W}}$ possesses an IR-divergent contribution in analogy to the corresponding wave-function renormalization constant in the conventional formalism [21, 22, 23].

IV THE EQUIVALENCE THEOREM

We have now all ingredients to derive the ET. The longitudinal polarization vector of a massive gauge boson ($V_a = W, Z$) with momentum k can be decomposed as

$$
\varepsilon^\mu_{a,\mathrm{L}}(k) = \frac{k^\mu}{M_a} + v^\mu_a(k), \qquad v^\mu_a(k) = \mathcal{O}\left(\frac{M_a}{k^0}\right),
\tag{30}
$$

i.e. its leading part at high energies is proportional to the momentum. Inserting this decomposition into the expressions (19) and (23) for the S-matrix elements, the Ward identities (18) directly imply for one external longitudinal gauge boson

$$
\begin{aligned}
\langle \ldots |S| W^\pm_{\mathrm{L}}(k) \ldots \rangle &= \pm R^{1/2}_{\hat{W}} G_{\hat{\phi}^\pm \ldots} + v^\mu_W(k) R^{1/2}_{\hat{W}} G_{\hat{W}^\pm \ldots},\\
\langle \ldots |S| Z_{\mathrm{L}}(k) \ldots \rangle &= \mathrm{i} R^{1/2}_{\hat{Z}\hat{Z}} G_{\hat{\chi}\ldots} + v^\mu_Z(k) R^{1/2}_{\hat{Z}\hat{Z}} G_{\hat{Z}\ldots} + v^\mu_Z(k) R^{1/2}_{\hat{A}\hat{Z}} G_{\hat{A}\ldots}.
\end{aligned}
\tag{31}
$$

Since unitarity ensures that S-matrix elements in the SM do not grow with powers of the gauge-boson energy k^0 in the high-energy limit, the contributions of $v^\mu_a(k)$ are of order $\mathcal{O}(M_a/k^0)$ and thus negligible,

$$
\begin{aligned}
\langle \ldots |S| W^\pm_{\mathrm{L}}(k) \ldots \rangle &= \pm R^{1/2}_{\hat{W}} G_{\hat{\phi}^\pm \ldots} + \mathcal{O}\left(\frac{M_{\mathrm{W}}}{k^0}\right),\\
\langle \ldots |S| Z_{\mathrm{L}}(k) \ldots \rangle &= \mathrm{i} R^{1/2}_{\hat{Z}\hat{Z}} G_{\hat{\chi}\ldots} + \mathcal{O}\left(\frac{M_{\mathrm{Z}}}{k^0}\right).
\end{aligned}
\tag{32}
$$

Equations (32) represent the precise formulation of the ET for the SM within the framework of the BFM.

The case of more longitudinal gauge bosons can be treated in the same way as in Ref. [2], the only difference being the factors R. As is easily seen, for each external longitudinal $W^\pm$ boson an extra factor $R^{1/2}_{\hat{W}}$ and for each external longitudinal Z boson an extra factor $R^{1/2}_{\hat{Z}\hat{Z}}$ has to be introduced. This concludes the derivation of the ET for the SM in the BFM.

The form (32) of the ET clearly displays one of the advantages of its formulation within the framework of the BFM. The correction factors, which modify the naïve form of the ET, are simply given by the residues of the renormalized massive gauge-boson propagators, which are needed for the calculation of S-matrix elements anyhow.

As pointed out in Sect. II, the underlying Ward identities (11) and (18) are not only valid order by order in perturbation theory but also after Dyson resummation. Consequently, also *the ET (32) within the BFM is valid after Dyson summation*.

The contributions of v^μ vanish owing to unitarity only if the energy E of the physical process is large compared to all masses present in the SM including the Higgs-boson mass, M_{H}. However, it is often very interesting to know to which order in $E^n M_{\mathrm{H}}^m$ the ET (32) is still valid also for large M_{H} or how it has to be modified in this case. This can be determined by a power-counting method developed in Refs. [8, 9]. Although this method was worked out for the conventional formalism, it is also applicable in the BFM since the leading powers in propagators and couplings are identical in both approaches.

The explicit expressions for the factors R in the SM in the BFM at one-loop order contain no terms of order $M_{\mathrm{H}}^2/M_{\mathrm{W}}^2$ for a large Higgs-boson mass. As a consequence these factors can be put to one in this limit if one is only interested in terms which are enhanced by powers of E^2/M_{W}^2 or $M_{\mathrm{H}}^2/M_{\mathrm{W}}^2$. For $M_{\mathrm{H}} \gg M_{\mathrm{W}}$ the factors R get $\log M_{\mathrm{H}}$ corrections, which are explicitly given in App. C.

V NON-LINEARLY REALIZED HIGGS SECTOR OF THE STANDARD MODEL

In the previous sections we have dealt with the commonly used *linear* realization of the Higgs sector of the SM, where the scalar Higgs doublet Φ is represented as

$$\Phi = \frac{1}{\sqrt{2}}\left((v+H)\mathbf{1} + \mathrm{i}\phi^a\tau^a\right), \qquad \phi^\pm = \frac{1}{\sqrt{2}}\left(\phi^2 \pm \mathrm{i}\phi^1\right), \qquad \chi = -\phi^3. \tag{33}$$

In (33) we have adopted the matrix notation of Ref. [24] with τ^a denoting the Pauli matrices. In the linear representation the physical Higgs field H is not gauge-invariant. Alternatively, the scalar field Φ can be *non-linearly* represented [25] as

$$\Phi = \frac{1}{\sqrt{2}}(v+H)U \qquad \text{with} \quad U = \exp\left(\frac{\mathrm{i}}{v}\phi^a\tau^a\right), \tag{34}$$

where the would-be Goldstone-boson fields ϕ^a parameterize the unitary matrix U. The explicit parameterization of U is not uniquely determined but the above exponential form is very convenient. The non-linear realization has the interesting property that the physical Higgs field H is gauge-invariant and that the scalar self-couplings do not involve the unphysical would-be Goldstone-boson fields but only H.

The application of the BFM to the non-linear realization of the Higgs sector (together with the corresponding gauge-invariant renormalization) was worked out in Ref. [24] and also briefly discussed in Ref. [17]. In the BFM approach the main difference between linear and non-linear realization relies in the splitting of the would-be Goldstone-boson fields into background and quantum parts: the unitary matrix U is split multiplicatively ($U \to \hat{U}U$). Using the above treatment of the linear realization as guideline, the ET for

the non-linear realization can be derived exactly in the same way. The actual calculation degenerates to a straightforward exercise so that it suffices to briefly describe the single steps.

The starting point is the derivation of the Ward identities, which follow from the invariance of the BFM effective action under background gauge transformations, as expressed by (3). As far as the Ward identities are concerned, the only difference between linear and non-linear realization lies in the explicit form of the gauge transformations of the background scalar fields $\hat{H}$, $\hat{\phi}^{\pm}$, $\hat{\chi}$. For the linear realization (33) they are explicitly given by Eq. (21) of Ref. [15], for the non-linear one (34) they can be deduced from Ref. [24]:

$$\begin{aligned}
\delta\hat{H} &= 0, \\
\delta\hat{\phi}^{\pm} &= \pm \mathrm{i} M_{\mathrm{W}}\delta\hat{\theta}^{\pm} - \frac{e}{2s_{\mathrm{W}}}\hat{\chi}\delta\hat{\theta}^{\pm} \mp \mathrm{i}e\hat{\phi}^{\pm}\left(\delta\hat{\theta}^{A} - \frac{c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2}{2c_{\mathrm{W}}s_{\mathrm{W}}}\delta\hat{\theta}^{Z}\right) \\
&\quad + g\left(\frac{\hat{\phi}^a\hat{\phi}^a}{v^2}\right)\frac{e^2}{4s_{\mathrm{W}}^2 M_{\mathrm{W}}}\left[\pm\mathrm{i}\hat{\phi}^{\pm}\hat{\phi}^{\pm}\delta\hat{\theta}^{\mp} \pm \mathrm{i}\hat{\phi}^{+}\hat{\phi}^{-}\delta\hat{\theta}^{\pm} \pm \mathrm{i}\hat{\chi}^2\delta\hat{\theta}^{\pm} + \frac{1}{c_{\mathrm{W}}}\hat{\phi}^{\pm}\hat{\chi}\delta\hat{\theta}^{Z}\right], \\
\delta\hat{\chi} &= -M_{\mathrm{Z}}\delta\hat{\theta}^{Z} + \frac{e}{2s_{\mathrm{W}}}(\hat{\phi}^{+}\delta\hat{\theta}^{-} + \hat{\phi}^{-}\delta\hat{\theta}^{+}) \\
&\quad + g\left(\frac{\hat{\phi}^a\hat{\phi}^a}{v^2}\right)\frac{e^2}{4s_{\mathrm{W}}^2 M_{\mathrm{W}}}\left[-\mathrm{i}\hat{\chi}(\hat{\phi}^{-}\delta\hat{\theta}^{+} - \hat{\phi}^{+}\delta\hat{\theta}^{-}) - \frac{2}{c_{\mathrm{W}}}\hat{\phi}^{+}\hat{\phi}^{-}\delta\hat{\theta}^{Z}\right].
\end{aligned} \tag{35}$$

Owing to the non-linearity of $\hat{\Phi}$, the gauge variations $\delta\hat{\phi}^{\pm}$ and $\delta\hat{\chi}$ involve arbitrary powers of $\hat{\phi}^{\pm}$ and $\hat{\chi}$ occurring in the function

$$g(x) = \frac{\cot(\sqrt{x})}{\sqrt{x}} - \frac{1}{x} = -\frac{1}{3} - \frac{x}{45} + \cdots \quad \text{with} \quad x = \frac{\hat{\phi}^a\hat{\phi}^a}{v^2} = \frac{e^2}{4s_{\mathrm{W}}^2 M_{\mathrm{W}}^2}(2\hat{\phi}^{+}\hat{\phi}^{-} + \hat{\chi}^2). \tag{36}$$

The g-independent terms in (35) are equal to the gauge variations in the linear representation with the physical Higgs field $\hat{H}$ omitted. The procedure of fixing the gauge of the background fields and performing the Legendre transformation to the generating functional Z_{c} of connected Green functions is carried out as above; in particular, (3) – (6) and (8) – (10) apply literally. In analogy to the derivation of (11) one obtains the functional form of the Ward identities for connected Green functions, which in contrast to (11) involves arbitrary powers of $(\delta Z_{\mathrm{c}}/\delta J_{\hat{\phi}^{\pm}})$ and $(\delta Z_{\mathrm{c}}/\delta J_{\hat{\chi}})$. However, the Ward identities (14) for the (renormalized) two-point functions are identical in both realizations. Moreover, the identities (15) for on-shell Green functions and their generalization for more gauge-fixing legs follow by the same reasoning as described in App. B implying (18) for on-shell amputated Green functions. Obviously, the wave-function renormalization and the construction of the S matrix, as described in Sect. III, do not rely on a specific property of the scalar sector. In summary, we arrive again at the ET in the form of (32).

It is quite easy to see that the explicit representation of the Higgs field Φ and its behavior under background gauge transformations in general are not important for the basic Ward identities (15)–(18). The only relevant terms in the gauge transformation of the would-be Goldstone-boson fields are the constant contributions, i.e. the ones which do not depend on the fields ϕ^a. Nevertheless, we have given the explicit form (35) of the

background gauge transformations for the scalar fields corresponding to the non-linear realization (34) in order to illustrate some interesting features. Comparing linear and non-linear realizations, one can see that all Ward identities that depend on the gauge-fixing term of the photon are identical in both cases, as the variations of the scalar fields with $\delta\hat{\theta}^A$ coincide. If gauge-fixing terms of the massive gauge fields are involved, the Ward identities in general are different. Moreover, the gauge invariance of $\hat{H}$ implies that neither $J_{\hat{H}}$ nor $\delta Z_{\mathrm{c}}/\delta J_{\hat{H}}$ occur in the functional form [the analogue of (11)] of the Ward identities. This means that all external Higgs-boson legs result from derivatives with respect to physical particles and occur in the same way (i.e. with the same field points and momenta) in each term of a given Ward identity.

Finally, we comment on the limit of a large Higgs-boson mass in the SM which is most conveniently discussed in the framework of the non-linear realization. Since in this formulation the scalar self interactions, which become strong for a heavy Higgs boson, are independent of the would-be Goldstone-boson fields, the SM formally reduces to the so-called *gauged non-linear σ-model* (GNLSM)[3][27]. The Lagrangian of the (non-renormalizable) GNLSM follows from the SM Lagrangian with non-linear scalar realization simply by disregarding the physical Higgs field H. Thus, omitting all terms (and remarks) involving H in this section, the results for the SM with non-linearly realized scalar sector transfer to the GNLSM. In particular, the basic Ward identities (18) and the construction of physical gauge-boson fields remain valid. However, since the GNLSM does not respect unitarity, the terms originating from v_a^μ in (31) do in general not vanish at high energies. The range of validity of the ET (32) is modified. This range can, for instance, be determined by applying power counting to the single Feynman graphs as proposed in Ref. [9] both for the GNLSM as well as for the SM with non-linear scalar realization and arbitrary M_{H}.

VI SUMMARY AND CONCLUSIONS

For the conventional approach for quantizing gauge fields, it is a well-known fact that Dyson summation in general spoils the underlying gauge symmetry in finite orders of perturbation theory, i.e. at the level of Green functions or S-matrix elements the Ward identities are violated. Consequently, gauge cancelations for high-energetic longitudinal gauge bosons, and in particular the validity of the Goldstone-boson equivalence theorem (ET), in general are disturbed when Dyson resummation is applied. We have explicitly derived and analyzed the Ward identities for connected Green functions within the background-field method (BFM) and found that the above-mentioned problems do not occur in this approach. It turns out that *BFM Ward identities for connected Green functions are exactly valid loop order by loop order in perturbation theory even after Dyson resummation* if the inverse propagators, i.e. the two-point vertex functions, are evaluated in the same loop order as all other vertex functions. In the same way *the ET within the BFM is valid after Dyson summation.*

As frequently discussed in the literature, within the conventional formalism the formulation of the ET in higher orders is non-trivial and requires the inclusion of correction

[3]The difference between the GNLSM and the SM for $M_{\mathrm{H}} \to \infty$ is of $\mathcal{O}(M_{\mathrm{H}}^{-2})$ at tree level. However, already at one-loop order differences of $\mathcal{O}(\log M_{\mathrm{H}})$ and $\mathcal{O}(1)$ exist, which can be quantified by an effective Lagrangian [24, 26].

factors which depend on the renormalization scheme of the unphysical sector. In view of this, we have analyzed the ET within the BFM in detail. Starting from the gauge invariance of the background-field effective action, we have derived Ward identities for on-shell amputated Green functions with arbitrary insertions of gauge-fixing terms. Using these Ward identities, which are formally equivalent to those in the conventional formalism, and a careful amputation procedure, we have derived the ET. The BFM simplifies, in particular, the amputation procedure and thus the precise formulation of the ET in higher orders. The correction factors that modify the naïve form of the ET are given by the residues of the gauge-boson propagators which are needed for the calculation of the S-matrix elements anyhow.

We have argued that our formulation of the ET is independent of the parameterization of the Higgs sector. This has been explicitly confirmed for a non-linear realization of the Higgs sector. In this context, we have also discussed the validity of the ET in the heavy Higgs-boson limit of the SM and the gauged non-linear σ-model. The power-counting methods needed in these cases to assess the range of validity of the ET can be directly transferred from the conventional formalism to the BFM.

As in previous applications, the BFM turns out to be superior to the conventional formalism. The above-mentioned advantages can be traced back to the gauge invariance of the background-field effective action and the associated Ward identities.

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APPENDIX

A MOMENTUM-SPACE AND FIELD CONVENTIONS

In order to avoid confusion, we summarize our conventions for the Fourier transformation needed for the transition from coordinate to momentum space. In this section all fields are generically denoted by ϕ. We start by defining the Fourier transform of a general vertex function:

$$\Gamma^{\phi_1\cdots\phi_n}(x_1,\ldots,x_n) = \int \frac{\mathrm{d}^4k_1}{(2\pi)^4}\cdots\int \frac{\mathrm{d}^4k_n}{(2\pi)^4}\,\exp\Bigl\{\mathrm{i}\sum_{l=1}^{n} k_l x_l\Bigr\}\,\tilde{\Gamma}^{\phi_1\cdots\phi_n}(k_1,\ldots,k_n). \tag{A1}$$

In $\tilde{\Gamma}^{\phi_1\cdots\phi_n}(k_1,\ldots,k_n)$ all momenta are incoming, and usually a δ-function for total momentum conservation is split off,

$$\tilde{\Gamma}^{\phi_1\cdots\phi_n}(k_1,\ldots,k_n) = (2\pi)^4\delta^{(4)}\Bigl(\sum_l k_l\Bigr)\,\Gamma^{\phi_1\cdots\phi_n}(k_1,\ldots,k_n). \tag{A2}$$

Transforming the fields via

$$\phi_l(x) = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\,\exp\left\{\mathrm{i}k_l x_l\right\}\,\phi_l(k), \tag{A3}$$

the generating functional for the vertex functions, the effective action, possesses the two representations

$$\begin{aligned}\Gamma[\phi] &= \sum_n \frac{1}{n!} \sum_{\phi_1,\ldots,\phi_n} \int \mathrm{d}^4x_1\, \phi_1(x_1) \cdots \int \mathrm{d}^4x_n\, \phi_n(x_n)\, \Gamma^{\phi_1\cdots\phi_n}(x_1,\ldots,x_n),\\ &= \sum_n \frac{1}{n!} \sum_{\phi_1,\ldots,\phi_n} \int \frac{\mathrm{d}^4k_1}{(2\pi)^4}\, \phi_1(-k_1) \cdots \int \frac{\mathrm{d}^4k_n}{(2\pi)^4}\, \phi_n(-k_n)\, \tilde{\Gamma}^{\phi_1\cdots\phi_n}(k_1,\ldots,k_n). \end{aligned} \tag{A4}$$

Because of the different integration measures $\int \mathrm{d}^4x$ and $\int \mathrm{d}^4k/(2\pi)^4$ it is natural to define the functional derivatives to extract the vertex functions in coordinate and momentum space as

$$\frac{\delta}{\delta f(x)} f(y) = \delta^{(4)}(x-y), \qquad \frac{\delta}{\delta f(p)} f(k) = (2\pi)^4 \delta^{(4)}(p-k). \tag{A5}$$

The transition to connected Green functions is performed via the Legendre transformation

$$\mathrm{i}J_{\phi_l^\dagger}(x) = -\frac{\delta\Gamma}{\delta\phi_l(x)}, \quad \phi_l(x) = \frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\phi_l^\dagger}(x)}, \quad Z_\mathrm{c}[J_\phi] = \Gamma[\phi] + \mathrm{i}\sum_l \int \mathrm{d}^4x\, J_{\phi_l^\dagger}(x)\phi_l(x). \tag{A6}$$

Transforming the sources via

$$J_{\phi_l}(x) = \int \frac{\mathrm{d}^4k}{(2\pi)^4} \exp\{\mathrm{i}k_l x_l\}\; J_{\phi_l}(k), \tag{A7}$$

the Legendre transformation takes the following form in momentum space:

$$\mathrm{i}J_{\phi_l^\dagger}(-k) = -\frac{\delta\Gamma}{\delta\phi_l(k)}, \quad \phi_l(k) = \frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\phi_l^\dagger}(-k)}, \quad Z_\mathrm{c}[J_\phi] = \Gamma[\phi] + \mathrm{i}\sum_l \int \frac{\mathrm{d}^4k}{(2\pi)^4}\, J_{\phi_l^\dagger}(-k)\phi_l(k). \tag{A8}$$

This leads to the two representations for the generating functional Z_c

$$\begin{aligned}Z_\mathrm{c}[\phi] &= \sum_n \frac{\mathrm{i}^n}{n!} \sum_{\phi_1,\ldots,\phi_n} \int \mathrm{d}^4x_1\, J_{\phi_1}(x_1) \cdots \int \mathrm{d}^4x_n\, J_{\phi_n}(x_n)\, G^{\phi_1\cdots\phi_n}(x_1,\ldots,x_n),\\ &= \sum_n \frac{\mathrm{i}^n}{n!} \sum_{\phi_1,\ldots,\phi_n} \int \frac{\mathrm{d}^4k_1}{(2\pi)^4}\, J_{\phi_1}(-k_1) \cdots \int \frac{\mathrm{d}^4k_n}{(2\pi)^4}\, J_{\phi_n}(-k_n)\, \tilde{G}^{\phi_1\cdots\phi_2}(k_1,\ldots,k_n). \end{aligned} \tag{A9}$$

Consequently, the *connected* Green functions $G^{\phi_1\cdots\phi_n}(x_1,\ldots,x_n)$ transform via

$$G^{\phi_1\cdots\phi_n}(x_1,\ldots,x_n) = \int \frac{\mathrm{d}^4k_1}{(2\pi)^4} \cdots \int \frac{\mathrm{d}^4k_n}{(2\pi)^4} \exp\Bigl\{\mathrm{i}\sum_{l=1}^n k_l x_l\Bigr\}\, \tilde{G}^{\phi_1\cdots\phi_n}(k_1,\ldots,k_n), \tag{A10}$$

where the momenta k_l are incoming. Again usually the δ-function for total momentum conservation is split off,

$$\tilde{G}^{\phi_1\cdots\phi_n}(k_1,\ldots,k_n) = (2\pi)^4 \delta^{(4)}\Bigl(\sum_l k_l\Bigr)\, G^{\phi_1\cdots\phi_n}(k_1,\ldots,k_n). \tag{A11}$$

In (11) both explicit sources $J_{\phi_l}(x)$ and the corresponding derivatives $\delta/\delta J_{\phi_l}(x)$ occur. The transition to momentum space is performed by applying $\int \mathrm{d}^4x\, \exp\{-\mathrm{i}kx\}\cdots$ to (11), where k is the momentum flowing into the considered field point x. This transforms the different terms as follows

$$\int \mathrm{d}^4x\, \exp\{-\mathrm{i}kx\}\, \frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_\phi(x)} = \frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_\phi(-k)}, \tag{A12}$$

$$\int \mathrm{d}^4x\, \exp\{-\mathrm{i}kx\}\, J_{\phi_1}(x)\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\phi_2}(x)} = \int \frac{\mathrm{d}^4q}{(2\pi)^4}\, J_{\phi_1}(q)\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\phi_2}(q-k)}, \tag{A13}$$

$$\int \mathrm{d}^4x\, \exp\{-\mathrm{i}kx\}\, \frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\phi_1}(x)}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\phi_2}(x)} = \int \frac{\mathrm{d}^4q}{(2\pi)^4}\, \frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\phi_1}(q-k)}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\phi_2}(-q)}. \tag{A14}$$

The fields that label the vertex and Green functions are defined to be incoming. For clarity, we list the relations between two-point functions and propagators:

$$\begin{aligned}
\Gamma^{\phi\phi^\dagger}(x,y) &= \frac{\delta^2\Gamma}{\delta\phi(x)\delta\phi^\dagger(y)}, \\
G^{\phi\phi^\dagger}(x,y) &= \frac{\delta^2 Z_\mathrm{c}}{\mathrm{i}\delta J_\phi(x)\mathrm{i}\delta J_{\phi^\dagger}(y)} = \langle 0|\mathcal{T}\phi^\dagger(x)\phi(y)|0\rangle_\mathrm{connected}, \\
\int \mathrm{d}^4y\, \Gamma^{\phi\phi^\dagger}(x,y)\, G^{\phi\phi^\dagger}(y,z) &= -\delta(x-z), \\
\Gamma^{\phi\phi^\dagger}(k,-k)\, G^{\phi\phi^\dagger}(k,-k) &= -1,
\end{aligned} \tag{A15}$$

where the field ϕ is assumed to mix with no other fields.

B PROOF OF THE WARD IDENTITIES FOR ON-SHELL AMPUTATED GREEN FUNCTIONS

B.1 Preliminaries

In order to proof the Ward identities for on-shell amputated Green functions, it is useful to rewrite the identities (11). Introducing the operators

$$\begin{aligned}
O_A(x) &= \mathrm{i}\partial_x^\mu \frac{\delta}{\mathrm{i}\delta J^\mu_{\hat{A}}(x)}, \qquad O_Z(x) = \mathrm{i}\partial_x^\mu \frac{\delta}{\mathrm{i}\delta J^\mu_{\hat{Z}}(x)} - \mathrm{i}\hat{\xi}_Z M_\mathrm{Z}\frac{\delta}{\mathrm{i}\delta J_{\hat{\chi}}(x)}, \\
O_\pm(x) &= \mathrm{i}\partial_x^\mu \frac{\delta}{\mathrm{i}\delta J^\mu_{\hat{W}^\pm}(x)} \mp \hat{\xi}_W M_\mathrm{W}\frac{\delta}{\mathrm{i}\delta J_{\hat{\phi}^\pm}(x)},
\end{aligned} \tag{B1}$$

which appear in all terms that result from the gauge-fixing term, we can arrange the Ward identities as

$$\begin{aligned}
\frac{1}{\hat{\xi}_A}\Box O_A Z_\mathrm{c} &= -\mathrm{i}\partial J_{\hat{A}} + e\sum_f Q_f\left(\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_f}J_f + J_{\bar{f}}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\bar{f}}}\right) \\
&\quad + \sum_\pm \frac{(\mp e)}{\hat{\xi}_W}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J^\mu_{\hat{W}^\mp}}\left(\mathrm{i}\partial^\mu O_\pm Z_\mathrm{c} - \hat{\xi}_W J^\mu_{\hat{W}^\mp}\right) + \sum_\pm e\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\hat{\phi}^\mp}}\left(M_\mathrm{W} O_\pm Z_\mathrm{c} \pm J_{\hat{\phi}^\mp}\right), \\
\frac{1}{\hat{\xi}_Z}(\Box + \hat{\xi}_Z M_\mathrm{Z}^2) O_Z Z_\mathrm{c} &= -\mathrm{i}\partial J_{\hat{Z}} - \mathrm{i}M_\mathrm{Z} J_{\hat{\chi}}
\end{aligned}$$

$$
\begin{aligned}
&- e\sum_f\Big(\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_f}(v_f+a_f\gamma_5)J_f + J_{\bar f}(v_f-a_f\gamma_5)\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\bar f}}\Big) + \frac{\mathrm{i}e}{2c_\mathrm{W}s_\mathrm{W}}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\hat\chi}}J_{\hat H}\\
&- \frac{e}{2c_\mathrm{W}s_\mathrm{W}}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\hat H}}\left(M_\mathrm{Z}O_Z Z_\mathrm{c} + \mathrm{i}J_{\hat\chi}\right) - \sum_\pm e\frac{c_\mathrm{W}^2-s_\mathrm{W}^2}{2c_\mathrm{W}s_\mathrm{W}}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\hat\phi^\mp}}\left(M_\mathrm{W}O_\pm Z_\mathrm{c} \pm J_{\hat\phi^\mp}\right),\\
&+ \sum_\pm\frac{(\pm e)}{\hat\xi_W}\frac{c_\mathrm{W}}{s_\mathrm{W}}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J^\mu_{\hat W^\mp}}\left(\mathrm{i}\partial^\mu O_\pm Z_\mathrm{c} - \hat\xi_W J^\mu_{\hat W^\mp}\right),
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{\hat\xi_W}&(\Box + \hat\xi_W M_\mathrm{W}^2)O_\pm Z_\mathrm{c} = -\mathrm{i}\partial J_{\hat W^\mp} \mp M_\mathrm{W} J_{\hat\phi^\mp}\\
&- \frac{e}{\sqrt{2}s_\mathrm{W}}\sum_{(f_+,f_-)}\Big(\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{f_\pm}}\frac{1+\gamma_5}{2}J_{f_\mp} + J_{\bar f_\pm}\frac{1-\gamma_5}{2}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\bar f_\mp}}\Big) \pm \frac{e}{2s_\mathrm{W}}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\hat\phi^\pm}}J_{\hat H}\\
&+ \frac{e}{2s_\mathrm{W}}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\hat\phi^\pm}}\left(M_\mathrm{Z}O_Z Z_\mathrm{c} + \mathrm{i}J_{\hat\chi}\right) - \frac{e}{2s_\mathrm{W}}\Big(\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\hat H}} \pm \mathrm{i}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J_{\hat\chi}}\Big)\left(M_\mathrm{W}O_\pm Z_\mathrm{c} \pm J_{\hat\phi^\mp}\right)\\
&\mp \frac{e}{\hat\xi_A}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J^\mu_{\hat W^\pm}}\left(\mathrm{i}\partial^\mu O_A Z_\mathrm{c} - \hat\xi_A J^\mu_{\hat A}\right) \pm \frac{e}{\hat\xi_Z}\frac{c_\mathrm{W}}{s_\mathrm{W}}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J^\mu_{\hat W^\pm}}\left(\mathrm{i}\partial^\mu O_Z Z_\mathrm{c} - \hat\xi_Z J^\mu_{\hat Z}\right)\\
&\pm \frac{e}{\hat\xi_W}\Big(\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J^\mu_{\hat A}} - \frac{c_\mathrm{W}}{s_\mathrm{W}}\frac{\delta Z_\mathrm{c}}{\mathrm{i}\delta J^\mu_{\hat Z}}\Big)\left(\mathrm{i}\partial^\mu O_\pm Z_\mathrm{c} - \hat\xi_W J^\mu_{\hat W^\mp}\right). \qquad \text{(B2)}
\end{aligned}
$$

The left-hand sides of (B2) represent the $\delta Z_\mathrm{c}/\delta J$ terms of (11); on the right-hand side the $(\delta Z_\mathrm{c}/\delta J)^2$ terms are combined with the $J_X\delta Z_\mathrm{c}/\delta J$ terms with X denoting a gauge or a would-be Goldstone-boson field.

For convenience we write (B2) generically,

$$
\begin{aligned}
\frac{1}{\hat\xi_a}&(\Box + \hat\xi_a M_a^2)O_a Z_\mathrm{c} = -\mathrm{i}\partial J_{\hat V_a^\dagger} + c_a J_{\hat\phi_a^\dagger} + \langle J_{f,\bar f,\hat H}\delta^1 Z_\mathrm{c}\rangle\\
&+ \sum_b\Big[\langle\delta^1 Z_\mathrm{c}\rangle\left(M_b^2 O_b Z_\mathrm{c} - c_b J_{\hat\phi_b^\dagger}\right) + \langle\delta^1 Z_\mathrm{c}\rangle\left(\mathrm{i}\partial^\mu O_b Z_\mathrm{c} - \hat\xi_b J^\mu_{\hat V_b^\dagger}\right)\Big], \qquad \text{(B3)}
\end{aligned}
$$

with $\hat\xi_\pm = \hat\xi_W$, $M_\pm = M_\mathrm{W}$ and

$$
O_a(x) = \mathrm{i}\partial^\mu_x\frac{\delta}{\mathrm{i}\delta J^\mu_{\hat V_a}(x)} + \hat\xi_a c_a\frac{\delta}{\mathrm{i}\delta J_{\hat\phi_a}(x)}. \qquad \text{(B4)}
$$

The index $a = Z, A, \pm$ refers also to the Ward identity according to (6). Of course, no would-be Goldstone-boson contribution appears for the photon. The symbol $\langle\delta^N Z_\mathrm{c}\rangle$ represents terms that do not involve explicit source factors but N derivatives of Z_c with respect to arbitrary sources. The symbol $\langle J_{f,\bar f,\hat H}\delta^N Z_\mathrm{c}\rangle$ stands for terms that involve in addition explicit source factors J_X where X is neither a gauge nor a would-be Goldstone-boson field. Application of an operator $O_{a_l}(x_l)$ to these terms results in

$$
O_{a_l}(x_l)\langle\delta^N Z_\mathrm{c}\rangle = \langle\delta^{N+1}Z_\mathrm{c}\rangle, \qquad O_{a_l}(x_l)\langle J_{f,\bar f,\hat H}\delta^N Z_\mathrm{c}\rangle = \langle J_{f,\bar f,\hat H}\delta^{N+1}Z_\mathrm{c}\rangle, \qquad \text{(B5)}
$$

because $O_{a_l}(x_l)$ only contains functional derivatives of gauge and would-be Goldstone-boson fields.

Transforming the operators $O_a(x)$ to momentum space, as specified in App. A, yields

$$
O_a(k) = -k^\mu\frac{\delta}{\mathrm{i}\delta J^\mu_{\hat V_a}(-k)} + \hat\xi_a c_a\frac{\delta}{\mathrm{i}\delta J_{\hat\phi_a}(-k)}. \qquad \text{(B6)}
$$

B.2 Exactly one gauge-fixing term

We want to show that the Ward identities (B3) imply

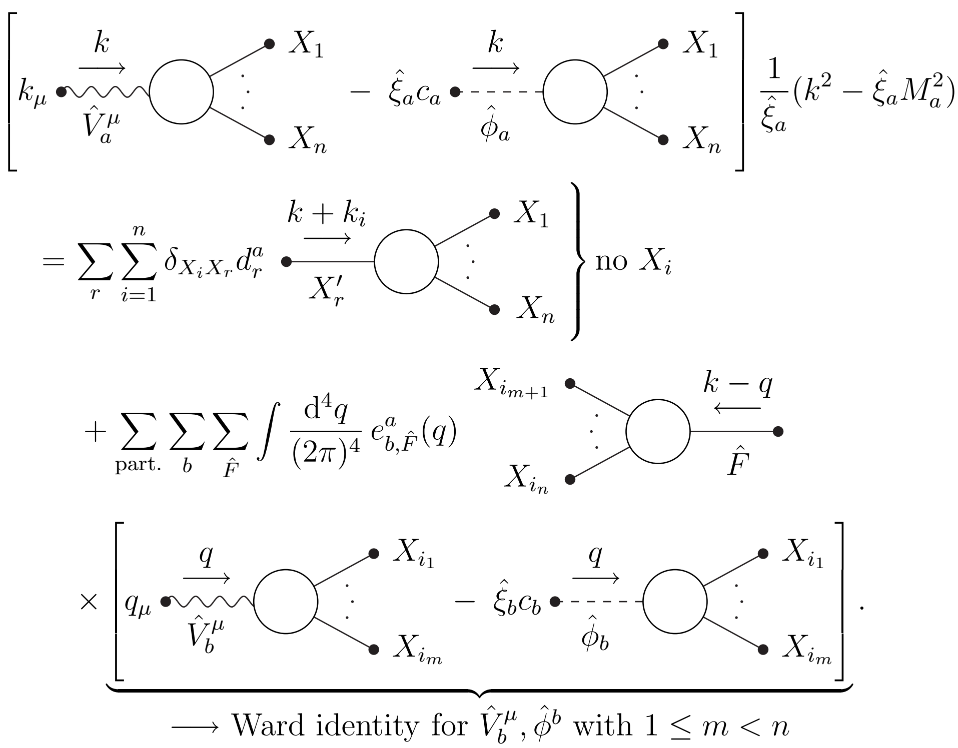
$$0 = \frac{1}{\hat{\xi}_a}(\Box + \hat{\xi}_a M_a^2) O_a(x) Z_{\mathrm{c}} \; + \text{ o.v.t.}\,, \tag{B7}$$

or in momentum space

$$0 = \frac{1}{\hat{\xi}_a}(k^2 - \hat{\xi}_a M_a^2) O_a(k) Z_{\mathrm{c}} \; + \text{ o.v.t.}\,, \tag{B8}$$

which implies (15) after dropping irrelevant factors. Again, o.v.t. (on-shell vanishing terms) stands for terms that vanish after taking derivatives with respect to physical fields and subsequent amputation and on-shell projection.

We consider the Ward identities resulting from (B3) by differentiating with respect to n physical fields X_i, $i = 1, \ldots, n \geq 2$, with incoming momenta k_i ($X_i = \hat{A}, \hat{Z}, \hat{W}^{\pm}, \hat{H}, f$). Their generic structure in momentum space is



(B9)

The $(n+1)$-point functions in the first line of (B9) correspond to the left-hand side of (B3). The second line of (B9) results from the terms of the form $d_r^a J_{X_r} \delta Z_{\mathrm{c}} / \delta J_{X_r'}$ summarized in $\langle J_{f,\bar{f},\hat{H}} \delta^1 Z_{\mathrm{c}} \rangle$ in the Ward identities (B3), where d_r^a denote constant factors. Because one of the derivatives $\delta/\delta J_{X_i}(-k_i)$ must act on the explicit source to produce a non-vanishing term, we obtain n-point functions with the incoming field $X_i = X_r$ with momentum k_i converted into the field X_r' with momentum $k + k_i$. The $\langle \delta^1 Z_{\mathrm{c}} \rangle O_b Z_{\mathrm{c}}$ terms in (B3) are all of the form $\int \mathrm{d}^4 q\, e^a_{b,\hat{F}}(q) \big(\delta Z_{\mathrm{c}} / \delta J_{\hat{F}}(q - k)\big) O_b(q) Z_{\mathrm{c}}$ in momentum space, in particular all of them involve a factor $O_b(q)$. Differentiating with respect to n external fields gives rise

to a convolution of Green functions as shown in the last two lines of (B9). The external fields are distributed in all possible ways over the two Green functions as indicated by $\sum_{\text{part.}}$. The sum over $\hat{F}$ runs over all fields that appear in the combination above and $e^a_{b,\hat{F}}(q)$ is a coefficient that possibly involves the momentum q. According to (A14), the incoming momentum k is distributed to both Green functions.

We now consider the case where the physical fields X_i are amputated, put on shell, and multiplied with the corresponding wave functions. The terms in the second line of (B9) do not possess a pole at $k_i^2 = m_{X_i}^2$ and thus vanish after amputation and going on shell with the X_i leg. Because all the Green functions in lines 3 and 4 must have at least one external X_i field (otherwise they vanish owing to $\delta Z_{\mathrm{c}}/\delta J_{\hat{F}}|_{J_{\hat{F}}=0} = 0$), the number m of X_i legs attached to the Green functions in the last line must be smaller than the total number n of X_i legs but bigger than zero, i.e. $1 \leq m < n$. But the last line is just the first line with fewer external legs. So (B3) implies that if the first line vanishes for all m with $1 \leq m < n$ it vanishes for n as well.

For $n = 2$ the expression in the last line of (B9) involves only two-point functions, and the physical fields X_i are either $\hat{H}$, $\hat{W}^\pm$, $\hat{A}$ or $\hat{Z}$. Using (14), we get zero for $\hat{H}$ and the momentum k_i^μ for a vector field. After contraction with the corresponding polarization vector $\varepsilon_\mu(k_i)$ this vanishes. Thus, the first line of (B9) vanishes for $n = 2$ and by induction for all n.

This proves (B7) or equivalently $\langle A|\mathcal{T} F_a(x)|B\rangle_{\text{connected}} = 0$.

B.3 General case

In order to prove $\langle A|\mathcal{T} F_{a_1}(x_1)\cdots F_{a_k}(x_k)|B\rangle_{\text{connected}} = 0$ for arbitrarily many insertions of gauge-fixing terms, or equivalently

$$0 = \left[\prod_{l=1}^{k} O_{a_l}(k_l)\right] Z_{\mathrm{c}} \,+\, \text{o.v.t.}\,, \tag{B10}$$

it is necessary to refine our recursive argumentation. For $k > 1$ the terms originating from $J\delta Z_{\mathrm{c}}/\delta J$ in general do not vanish separately after amputation and on-shell projection but only in combination with $(\delta Z_{\mathrm{c}}/\delta J)^2$ terms. This is due to terms that arise when the additional operators $O_{a_l}(k_l)$ act on the explicit source terms in (B3).

The Ward identities (B3) imply

$$\begin{aligned}
O_{a_l}(x_l)\left(M_a^2 O_a Z_{\mathrm{c}} - c_a J_{\hat{\phi}_a^\dagger}\right) &= \langle J_{f,\bar{f},\hat{H}}\delta^2 Z_{\mathrm{c}}\rangle \\
&+ \sum_b \Big[\Big(\langle\delta^2 Z_{\mathrm{c}}\rangle + \langle\delta^1 Z_{\mathrm{c}}\rangle O_{a_l}(x_l)\Big)\left(M_b^2 O_b Z_{\mathrm{c}} - c_b J_{\hat{\phi}_b^\dagger}\right) \\
&\quad + \Big(\langle\delta^2 Z_{\mathrm{c}}\rangle + \langle\delta^1 Z_{\mathrm{c}}\rangle O_{a_l}(x_l)\Big)\left(\mathrm{i}\partial^\mu O_b Z_{\mathrm{c}} - \hat{\xi}_b J^\mu_{\hat{V}_b^\dagger}\right)\Big]\,, \\
O_{a_l}(x_l)\left(\mathrm{i}\partial^\mu O_a Z_{\mathrm{c}} - \hat{\xi}_a J^\mu_{\hat{V}_a^\dagger}\right) &= \langle J_{f,\bar{f},\hat{H}}\delta^2 Z_{\mathrm{c}}\rangle \\
&+ \sum_b \Big[\Big(\langle\delta^2 Z_{\mathrm{c}}\rangle + \langle\delta^1 Z_{\mathrm{c}}\rangle O_{a_l}(x_l)\Big)\left(M_b^2 O_b Z_{\mathrm{c}} - c_b J_{\hat{\phi}_b^\dagger}\right) \\
&\quad + \Big(\langle\delta^2 Z_{\mathrm{c}}\rangle + \langle\delta^1 Z_{\mathrm{c}}\rangle O_{a_l}(x_l)\Big)\left(\mathrm{i}\partial^\mu O_b Z_{\mathrm{c}} - \hat{\xi}_b J^\mu_{\hat{V}_b^\dagger}\right)\Big]\,,
\end{aligned} \tag{B11}$$

as can be verified by inserting for O_aZ_c in the left-hand side (B3) and using (B5) and

$$\begin{aligned}
0 &= O_{a_l}(x_l)\left(M_a^2\hat{\xi}_a(\Box+\hat{\xi}_a M_a^2)^{-1}(-\mathrm{i}\partial J_{\hat{V}_a^\dagger}+c_a J_{\hat{\phi}_a^\dagger})-c_a J_{\hat{\phi}_a^\dagger}\right),\\
0 &= O_{a_l}(x_l)\left(\mathrm{i}\partial^\mu\hat{\xi}_a(\Box+\hat{\xi}_a M_a^2)^{-1}(-\mathrm{i}\partial J_{\hat{V}_a^\dagger}+c_a J_{\hat{\phi}_a^\dagger})-\hat{\xi}_a J^\mu_{\hat{V}_a^\dagger}\right).
\end{aligned} \tag{B12}$$

Applying $k-2$ operators $O_{a_l}(x_l)$ to (B11) leads to

$$\begin{aligned}
&\left[\prod_{l=2}^{k} O_{a_l}(x_l)\right]\left(M_a^2 O_a Z_\mathrm{c}-c_a J_{\hat{\phi}_a^\dagger}\right)=\langle J_{f,\bar{f},\hat{H}}\delta^k Z_\mathrm{c}\rangle\\
&\quad+\sum_b\sum_{i=0}^{k-1}\left[\langle\delta^{k-i}Z_\mathrm{c}\rangle\langle O^i\rangle\left(M_b^2 O_b Z_\mathrm{c}-c_b J_{\hat{\phi}_b^\dagger}\right)+\langle\delta^{k-i}Z_\mathrm{c}\rangle\langle O^i\rangle\left(\mathrm{i}\partial^\mu O_b Z_\mathrm{c}-\hat{\xi}_b J^\mu_{\hat{V}_b^\dagger}\right)\right],\\
&\left[\prod_{l=2}^{k} O_{a_l}(x_l)\right]\left(\mathrm{i}\partial^\mu O_a Z_\mathrm{c}-\hat{\xi}_a J^\mu_{\hat{V}_a^\dagger}\right)=\langle J_{f,\bar{f},\hat{H}}\delta^k Z_\mathrm{c}\rangle\\
&\quad+\sum_b\sum_{i=0}^{k-1}\left[\langle\delta^{k-i}Z_\mathrm{c}\rangle\langle O^i\rangle\left(M_b^2 O_b Z_\mathrm{c}-c_b J_{\hat{\phi}_b^\dagger}\right)+\langle\delta^{k-i}Z_\mathrm{c}\rangle\langle O^i\rangle\left(\mathrm{i}\partial^\mu O_b Z_\mathrm{c}-\hat{\xi}_b J^\mu_{\hat{V}_b^\dagger}\right)\right],
\end{aligned} \tag{B13}$$

where the symbol $\langle O^i\rangle$ stands for a product of i operators out of $\prod_l O_{a_l}(x_l)$. By taking $n\geq 0$ derivatives $\delta/\delta J_X$ for arbitrary physical fields X and setting all sources to zero one obtains Ward identities for Green functions from (B13).

In order to prove that the left-hand sides of (B13) are equal to zero after amputating the physical fields and putting them on shell, we use induction both in the number n of physical fields and in the number k of operators O_{a_l}. For $k=1$, i.e. if the factor $\prod_{l=2}^k O_{a_l}(x_l)$ is missing, (B13) reduces to (B3) apart from explicit source terms $J_{\hat{\phi}_a^\dagger}, J_{\hat{V}_a^\dagger}$, which do not contribute to on-shell Green functions. Consequently, the left-hand side of (B13) vanishes according to (B8) for $k=1$. If we assume that the statement holds for all $k<K$ all terms involving $\langle O^k\rangle$ with $k<K$ drop out in the Ward identity resulting from (B13) for on-shell amputated Green functions. Moreover, the terms $\langle J_{f,\bar{f},\hat{H}}\delta^k Z_\mathrm{c}\rangle$ do not contribute on-shell because they miss a pole for a physical particle as described in the previous section. Thus, the only terms that can yield non-vanishing contributions obey the recursion relations

$$\begin{aligned}
&\left[\prod_{l=2}^{K} O_{a_l}(x_l)\right]\left(M_a^2 O_a Z_\mathrm{c}-c_a J_{\hat{\phi}_a^\dagger}\right)=\\
&\quad\sum_b\left[\langle\delta^1 Z_\mathrm{c}\rangle\left[\prod_{l=2}^{K} O_{a_l}(x_l)\right]\left(M_b^2 O_b Z_\mathrm{c}-c_b J_{\hat{\phi}_b^\dagger}\right)+\langle\delta^1 Z_\mathrm{c}\rangle\left[\prod_{l=2}^{K} O_{a_l}(x_l)\right]\left(\mathrm{i}\partial^\mu O_b Z_\mathrm{c}-\hat{\xi}_b J^\mu_{\hat{V}_b^\dagger}\right)\right],\\
&\left[\prod_{l=2}^{K} O_{a_l}(x_l)\right]\left(\mathrm{i}\partial^\mu O_a Z_\mathrm{c}-\hat{\xi}_a J^\mu_{\hat{V}_a^\dagger}\right)=\\
&\quad\sum_b\left[\langle\delta^1 Z_\mathrm{c}\rangle\left[\prod_{l=2}^{K} O_{a_l}(x_l)\right]\left(M_b^2 O_b Z_\mathrm{c}-c_b J_{\hat{\phi}_b^\dagger}\right)+\langle\delta^1 Z_\mathrm{c}\rangle\left[\prod_{l=2}^{K} O_{a_l}(x_l)\right]\left(\mathrm{i}\partial^\mu O_b Z_\mathrm{c}-\hat{\xi}_b J^\mu_{\hat{V}_b^\dagger}\right)\right].
\end{aligned} \tag{B14}$$

In order to show that the left-hand side of (B14) vanishes upon taking n derivatives $\delta/\delta J_X$ with respect to physical fields X and setting $J=0$ for all fields, we again exploit the

recursive structure of (B14) and proceed by induction in n. For $n=0$ the statement is trivial owing to $\langle \delta^1 Z_{\rm c}\rangle|_{J=0}=0$. Taking $n>0$ derivatives $\delta/\delta J_X$ from the right-hand side of (B14) only those terms can possibly contribute where at least one derivative is applied to $\langle \delta^1 Z_{\rm c}\rangle$. Hence, the statement for n is traced back to the statement for $n-1$, which completes the induction. Thus, we have proved

$$\begin{aligned}
&\left[\prod_{l=2}^{k} O_{a_l}(x_l)\right]\left(M_a^2 O_a Z_{\rm c} - c_a J_{\hat{\phi}_a^\dagger}\right) = \text{o.v.t.}\,,\\
&\left[\prod_{l=2}^{k} O_{a_l}(x_l)\right]\left({\rm i}\partial^\mu O_a Z_{\rm c} - \hat{\xi}_a J^\mu_{\hat{V}_a^\dagger}\right) = \text{o.v.t.}\,. \qquad \text{(B15)}
\end{aligned}$$

For $k=2$ and $n=0$ we recover the Ward identities for the propagators (14). For $k+n>2$ the explicit source terms drop out and we obtain (B10) from the second equation of (B15), since the derivative ∂^μ translates to a simple momentum factor in momentum space, which can be dropped.

C EXPLICIT ONE-LOOP RESULTS FOR WAVE-FUNCTION NORMALIZATION FACTORS

In Sect. III we have described how to calculate S-matrix elements from background-field Green functions which are renormalized using the scheme of Ref. [15]. For external gauge bosons one needs the UV-finite wave-function renormalization constants $R_{\hat{W}}$, $R_{\hat{A}\hat{Z}}$, and $R_{\hat{Z}\hat{Z}}$. These can be determined from the transverse parts of the renormalized gauge-boson two-point functions (self-energies). In one-loop approximation, the bosonic contributions to the transverse parts of the BFM self-energies read in the 't Hooft–Feynman gauge, i.e. for $\xi_Q=1$,

$$\begin{aligned}
\Sigma'^{\hat{W}^+\hat{W}^-}_{\rm T}(M_{\rm W}^2)\Big|^{\rm bos}_{\xi_Q=1} ={}& \frac{\alpha}{4\pi s_{\rm w}^2}\Bigg[-\frac{1}{9}+\frac{1}{12}\left(\frac{M_{\rm H}^2}{M_{\rm W}^2}-1\right)^2 B_0(0,M_{\rm W},M_{\rm H})+\frac{2}{3}s_{\rm w}^2 B_0(0,0,M_{\rm W})\\
&+\frac{s_{\rm w}^4}{12c_{\rm w}^4}(1+8c_{\rm w}^2)B_0(0,M_{\rm W},M_{\rm Z})-\frac{M_{\rm H}^2}{12M_{\rm W}^2}\left(\frac{M_{\rm H}^2}{M_{\rm W}^2}-2\right)B_0(M_{\rm W}^2,M_{\rm W},M_{\rm H})\\
&-8s_{\rm w}^2 B_0(M_{\rm W}^2,M_{\rm W},0)-\frac{1}{12c_{\rm w}^4}\left(96c_{\rm w}^6-16c_{\rm w}^4+6c_{\rm w}^2+1\right)B_0(M_{\rm W}^2,M_{\rm W},M_{\rm Z})\\
&-4s_{\rm w}^2 M_{\rm W}^2 B_0'(M_{\rm W}^2,M_{\rm W},\lambda)+\left(\frac{M_{\rm H}^4}{12M_{\rm W}^4}-\frac{M_{\rm H}^2}{3M_{\rm W}^2}+1\right)M_{\rm W}^2 B_0'(M_{\rm W}^2,M_{\rm W},M_{\rm H})\\
&-\frac{1}{12c_{\rm w}^4}(4c_{\rm w}^2-1)\left(12c_{\rm w}^4+20c_{\rm w}^2+1\right)M_{\rm W}^2 B_0'(M_{\rm W}^2,M_{\rm W},M_{\rm Z})\Bigg]-2\delta Z_e-\frac{c_{\rm w}^2}{s_{\rm w}^2}\frac{\delta c_{\rm w}^2}{c_{\rm w}^2},
\end{aligned}$$

$$\begin{aligned}
\Sigma'^{\hat{Z}\hat{Z}}_{\rm T}(M_{\rm Z}^2)\Big|^{\rm bos}_{\xi_Q=1} ={}& \frac{\alpha}{4\pi c_{\rm w}^2 s_{\rm w}^2}\Bigg[\frac{1}{9}(1-2c_{\rm w}^2)+\frac{1}{12}\left(\frac{M_{\rm H}^2}{M_{\rm Z}^2}-1\right)^2 B_0(0,M_{\rm Z},M_{\rm H})\\
&-\frac{M_{\rm H}^2}{12M_{\rm Z}^2}\left(\frac{M_{\rm H}^2}{M_{\rm Z}^2}-2\right)B_0(M_{\rm Z}^2,M_{\rm Z},M_{\rm H})-\frac{1}{12}\left(84c_{\rm w}^4+4c_{\rm w}^2-1\right)B_0(M_{\rm Z}^2,M_{\rm W},M_{\rm W})\\
&-\frac{1}{12}(4c_{\rm w}^2-1)\left(12c_{\rm w}^4+20c_{\rm w}^2+1\right)M_{\rm Z}^2 B_0'(M_{\rm Z}^2,M_{\rm W},M_{\rm W})
\end{aligned}$$

$$+\left(\frac{M_{\mathrm{H}}^4}{12M_{\mathrm{Z}}^4}-\frac{M_{\mathrm{H}}^2}{3M_{\mathrm{Z}}^2}+1\right)M_{\mathrm{Z}}^2B_0'(M_{\mathrm{Z}}^2,M_{\mathrm{Z}},M_{\mathrm{H}})\Bigg]-2\delta Z_e-\frac{c_{\mathrm{W}}^2-s_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2},$$

$$\begin{aligned}\Sigma_{\mathrm{T}}^{\hat{A}\hat{Z}}(M_{\mathrm{Z}}^2)\Big|_{\xi_Q=1}^{\mathrm{bos}} &= \frac{\alpha}{4\pi c_{\mathrm{W}}s_{\mathrm{W}}}M_{\mathrm{Z}}^2\Bigg[\frac{1}{9}-\frac{2c_{\mathrm{W}}^2}{3}(6c_{\mathrm{W}}^2-1)B_0(0,M_{\mathrm{W}},M_{\mathrm{W}})\\ &\quad+\frac{1}{6}\left(24c_{\mathrm{W}}^4+38c_{\mathrm{W}}^2+1\right)B_0(M_{\mathrm{Z}}^2,M_{\mathrm{W}},M_{\mathrm{W}})\Bigg]+\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2}M_{\mathrm{Z}}^2.\end{aligned}\tag{C1}$$

We suppress the one-loop fermionic contributions to the self-energies since these are identical in the BFM and the conventional formalism and therefore can be simply inferred from the explicit results of Ref. [22]. In this reference also the scalar two-point function $B_0(p^2,m_0,m_1)$ and its momentum derivative $B_0'=\partial B_0/\partial p^2$ can be found. We note that $\Sigma_{\mathrm{T}}'^{\hat{W}^+\hat{W}^-}(M_{\mathrm{W}}^2)$, and thus $R_{\hat{W}}$, gets an IR-divergent contribution contained in

$$B_0'(M_{\mathrm{W}}^2,M_{\mathrm{W}},\lambda)=-\frac{1}{M_{\mathrm{W}}^2}\left[1+\log\left(\frac{\lambda}{M_{\mathrm{W}}}\right)\right],\tag{C2}$$

where λ denotes the infinitesimal photon mass used as IR regulator. The counterterms δZ_e and δc_{W}^2 read

$$\delta Z_e|^{\mathrm{bos}}=-\frac{\alpha}{4\pi}\left[\frac{7}{2}B_0(0,M_{\mathrm{W}},M_{\mathrm{W}})+2M_{\mathrm{W}}^2B_0'(0,M_{\mathrm{W}},M_{\mathrm{W}})\right],$$

$$\begin{aligned}\delta c_{\mathrm{W}}^2|^{\mathrm{bos}} &= \frac{\alpha}{4\pi}\Bigg[-\frac{1}{9}\left(36c_{\mathrm{W}}^4+24c_{\mathrm{W}}^2+1\right)\\ &\quad+\frac{1}{12s_{\mathrm{W}}^2}\left(\frac{M_{\mathrm{H}}^2}{M_{\mathrm{Z}}^2}-1\right)^2B_0(0,M_{\mathrm{Z}},M_{\mathrm{H}})-\frac{c_{\mathrm{W}}^2}{12s_{\mathrm{W}}^2}\left(\frac{M_{\mathrm{H}}^2}{M_{\mathrm{W}}^2}-1\right)^2B_0(0,M_{\mathrm{W}},M_{\mathrm{H}})\\ &\quad+\frac{2c_{\mathrm{W}}^2}{3}\left(6c_{\mathrm{W}}^2+1\right)B_0(0,0,M_{\mathrm{W}})+\frac{1}{12c_{\mathrm{W}}^2}\left(24c_{\mathrm{W}}^4-7c_{\mathrm{W}}^2-1\right)B_0(0,M_{\mathrm{Z}},M_{\mathrm{W}})\\ &\quad-4c_{\mathrm{W}}^2B_0(M_{\mathrm{W}}^2,M_{\mathrm{W}},0)+\frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\left(\frac{M_{\mathrm{H}}^4}{12M_{\mathrm{W}}^4}-\frac{M_{\mathrm{H}}^2}{3M_{\mathrm{W}}^2}+1\right)B_0(M_{\mathrm{W}}^2,M_{\mathrm{W}},M_{\mathrm{H}})\\ &\quad-\frac{1}{s_{\mathrm{W}}^2}\left(\frac{M_{\mathrm{H}}^4}{12M_{\mathrm{Z}}^4}-\frac{M_{\mathrm{H}}^2}{3M_{\mathrm{Z}}^2}+1\right)B_0(M_{\mathrm{Z}}^2,M_{\mathrm{Z}},M_{\mathrm{H}})\\ &\quad+\frac{4c_{\mathrm{W}}^2-1}{12s_{\mathrm{W}}^2}\left(12c_{\mathrm{W}}^4+20c_{\mathrm{W}}^2+1\right)\left(B_0(M_{\mathrm{Z}}^2,M_{\mathrm{W}},M_{\mathrm{W}})-\frac{1}{c_{\mathrm{W}}^2}B_0(M_{\mathrm{W}}^2,M_{\mathrm{W}},M_{\mathrm{Z}})\right)\Bigg]\end{aligned}\tag{C3}$$

in one-loop approximation. While the general background-field gauge-boson self-energies, and also the derivatives in (C1), depend on the quantum gauge parameter ξ_Q, the counterterms δZ_e and δc_{W}^2 are gauge-parameter-independent [15].

Finally, we compare the one-loop expressions for the wave-function renormalization factors R of the gauge bosons in the linear and non-linear realization of the Higgs sector. This can easily be done by inspecting the differences in the Feynman rules. All couplings of exactly one would-be Goldstone-boson field to any other fields are identical in both realizations so that possible differences at one loop could only originate from quartic couplings between two vector and two scalar fields. However, also these differences drop

out in the factors R so that the one-loop expressions for the R's coincide in the linear and non-linear realization.

We can directly exploit this coincidence when calculating the leading contributions to the factors R in the limit of a large Higgs-boson mass. To this end we use the one-loop effective Lagrangian of Ref. [24] which quantifies the difference between the non-linearly realized SM with a heavy Higgs boson and the GNLSM. At one loop the differences between SM and GNLSM in the above-mentioned self-energies read

$$\begin{aligned} \Sigma'^{\hat{W}^+\hat{W}^-}_{\mathrm{T}}(M_{\mathrm{W}}^2)\Big|^{\mathrm{SM-GNLSM}} &= \frac{\alpha}{48\pi s_{\mathrm{W}}^2} 11 \left[\Delta_{M_{\mathrm{H}}} + \frac{5}{6}\right] \;+\; \mathcal{O}(M_{\mathrm{W}}^2/M_{\mathrm{H}}^2), \\ \Sigma'^{\hat{Z}\hat{Z}}_{\mathrm{T}}(M_{\mathrm{Z}}^2)\Big|^{\mathrm{SM-GNLSM}} &= \frac{\alpha}{48\pi s_{\mathrm{W}}^2} \left(11 - 9\frac{s_{\mathrm{W}}^2}{c_{\mathrm{W}}^2}\right) \left[\Delta_{M_{\mathrm{H}}} + \frac{5}{6}\right] \;+\; \mathcal{O}(M_{\mathrm{W}}^2/M_{\mathrm{H}}^2), \\ \Sigma^{\hat{A}\hat{Z}}_{\mathrm{T}}(M_{\mathrm{Z}}^2)\Big|^{\mathrm{SM-GNLSM}} &= -M_{\mathrm{Z}}^2 \frac{5\alpha}{24\pi c_{\mathrm{W}} s_{\mathrm{W}}} \left[\Delta_{M_{\mathrm{H}}} + \frac{5}{6}\right] \;+\; \mathcal{O}(M_{\mathrm{W}}^2/M_{\mathrm{H}}^2), \end{aligned} \tag{C4}$$

where the $\log M_{\mathrm{H}}$ contributions are contained in the UV-divergent term $\Delta_{M_{\mathrm{H}}}$ defined by

$$\Delta_{M_{\mathrm{H}}} = \frac{2}{4-D} - \log\left(\frac{M_{\mathrm{H}}^2}{\mu^2}\right) - \gamma_{\mathrm{E}} + \log(4\pi). \tag{C5}$$

We have explicitly checked that the $\log M_{\mathrm{H}}$ terms of (C4) are in agreement with those obtained by a large-M_{H} expansion of (C1).

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